A DESIGN METHOD FOR PRISMATIC PRESTRESSED CONTINUOUS BOX GIRDER BRIDGES

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INTRODUCTION

Box girder bridges are often used in the span range of 30 to 200 m. They are quite popular due to inherent torsional rigidity that gives very good load sharing characteristics. The box girder consists of single or multiple cells with cantilever overhangs on either side at the top flange. Since these structures are generally constructed as continuous over the supports, the design process becomes a complicated task. This is especially true with prestressed concrete box girders due to the secondary moments induced due to prestressing effects.

The secondary moments are induced because there is a tendency for the continuous prestressed bridge to lift up at the supports which is prevented by the bearings. This uplift and as a consequence, the secondary moments depend on the prestressing forces applied and the tendon profile selected. In the normal design process for prestressed concrete, the prestressing forces and the tendon profile is selected on the basis of the bending moment envelope due to loads. However, for continuous bridges, the secondary moments alter the bending moment envelope, but these secondary moments are not known until the tendon profile and the prestressing forces are known. This leads to an iterative process which could be quite cumbersome since the amount of calculations involved is large.

A design method has been developed by Jayasinghe (1992) that minimises the amount of iteration and also allows the designer full control of the structural design process. In this method, the analysis was carried out by considering that the bridge can be modelled as a continuous beam. In this paper, it is shown that the same method can be extended to the grillage analysis thus allowing the designer to use grillage analysis for the structural design process. The main advantages of this method include the ability of the design engineer to select the secondary moments at the beginning of the design process and to achieve exactly those assumed without going through an iterative process, ability of the design engineer to optimise the section dimensions by altering the secondary moments appropriately, and the section of a prestressing force which is close to minimum throughout the length of the bridge thus achieving savings in cost. The tendon profile and the prestressing forces so selected can be further validated using a detailed design taking all the effects into account.

SELECTION OF CROSS SECTIONAL DIMENSIONS

The cross section of a box girder can be idealised as shown in Figure 1 for the preliminary design. The important parameters can be selected as given below.

Overall Depth of the Section

This is determined by the design engineer in general. It may be specified in the client's brief or it may be governed by vertical alignment considerations. If it is not given, the designer is free to select a reasonable value for the span/depth ratio; values between 14 and 25 are typical. It has been stated by Gee (1987) that it is economical to use larger depths and a reduced prestress if it is allowable. Greater depths allow a reduction of the bottom flange area, but carry the penalty of increased weight due to increased depth of web, which may outweigh the benefit from the improved flexural behaviour. The most economical solution can be obtained by using few alternative designs, that can easily be achieved with the design method described in this paper.

The Width of the Top Flange

When the top flange is used for the roadway in highway structures, the width is controlled by the number of traffic lanes, cycle lanes etc. Hence, the selection of top flange width can be easily selected by the design engineer.

The Web Spacing

The web spacing is governed by local bending of the top slab. Increasing the web spacing increases the thickness required for the top slab, but reduces the number of webs. Thus, there is a trade-off between additional material in the top slab and in the webs. However, extra material in the flanges contribute to global bending resistance and stiffness, whereas the extra web material simply add to the weight. The practical maximum web spacing for a reinforced concrete top slab is about 6.0 m and for a prestressed concrete top slab is about 7.5 m.

The Cantilever Overhang

In box girder bridges, cantilever overhangs are used which is a convenient way of minimising the number of webs while achieving the required width for the top flange. Generally the length of the cantilever overhang is kept around 0.5 times the spacing between the webs.

The Thickness of the Webs

The primary purpose of the webs is to resist the shear stresses due to torsional moments and shear forces. However, for constructable thicknesses of the webs, adequate shear resistance can be obtained by adding shear reinforcement, except possibly near the supports, when it becomes economic to increase the thickness of the webs locally. Thick webs carry a double penalty (Gee, 1987), since they add not only to the dead weight without resisting additional resistance, but also the area that has to be prestressed. Thus, the most logical criterion for selecting web thicknesses becomes one of constructability. The following guidelines have been suggested by Podolny & Muller (1982).

Table 1: Web Thicknesses Required Depending on the Type of Ducts

Type of ducts	Web thickness (m)
No prestressing ducts in webs	0.200
Small ducts for vertical prestressing	0.250
Ducts for the prestressing cables	0.300
Anchors for the prestressing cables	0.350

The Thickness of the Top Flange

The top flange thickness is generally governed by local bending considerations and the form of construction. The thicknesses for the top flange have been obtained from the survey by Swann (1972), where the thickness depends on the clear spacing between webs. Once these thicknesses are adopted, the top flange area would be large enough to meet the area required by the global bending criteria.

Table 2: Minimum Thicknesses Required for the Top Flange

Clear spacing between webs (m)	Thickness for transversely reinforced web (m)	Thickness for transversely prestressed web (m)
up to 3.00	0.250	0.200
3.00 - 4.50	0.300	0.225
4.50 - 6.00	0.350	0.275
6.00 - 7.50	-	0.300

The Width of the Bottom Flange

The designer turns out to have surprisingly little control over the bottom flange width. It depends on the cantilever overhang, inclination and number of webs, the clear spacing between webs and the thickness of webs. At this stage of the design process, the only decision that the designer needs to make is the inclination of the webs, which may be governed by the aesthetic considerations.

The Thickness of the Bottom Flange

In prismatic box girder bridges, the bottom flange provides the compressive resistance over the supports. Away from the supports, the function of the bottom flange is primarily non structural such as supporting the services etc. Therefore, it is advantageous to minimise the thickness of the bottom flange as much as possible. The minimum constructable thickness of the bottom flange is 125 mm, but this minimum value is not adopted due to difficulties of preventing cracks caused by horizontal shear. The recommended minimum thickness is 175 mm. However, this must be checked with the thickness required over the supports.

OPTIMISATION OF SECTION DIMENSIONS

In box girder bridges, it is useful to minimise the cross sectional area, thus minimising the dead load moments. The top flange dimensions are generally governed by the flexural behaviour in the transverse direction. These dimensions will generally be much larger than the dimensions required for longitudinal flexural behaviour and hence minimum should be selected by considering the transverse behaviour. The dimensions and number of webs must be minimised, since the shear is critical only close to the supports. However, the minimum thickness of webs is generally governed by the constructability hence cannot be minimised beyond that can be constructed. The bottom flange width depends on the inclination of webs, which may depend on aesthetics and ease of construction.

The thickness of the bottom flange is governed by the flexural behaviour in the longitudinal direction. In prismatic box girder bridges, the thickness of bottom flange over the support is governed by the hogging moments induced there. It is possible to change the magnitude of the hogging moments by using the secondary moments since they are generally of sagging nature. The advantage is that it is possible to use the minimum serviceable thickness of 175 mm by selecting an appropriate set of secondary moments thus minimising the moment due to dead load.

USE OF SECONDARY MOMENTS TO MINIMISE THE BOTTOM FLANGE THICKNESS

This can be achieved in the following manner:

1. Select the section dimensions using the guide lines given in Section 2.
2. Determine the dead load bending moment envelope. The dead load moments will form an envelope due to this reason. In continuous bridges, it is not possible to carry out the construction in one step. A large number of different construction techniques such as span by span, incremental launching, segmental construction with balance cantilever etc have often been used. Span by span construction is quite common for medium scale bridges where the prismatic box girders are used. In this method, one span and about 5 m length of the next span are constructed in one operation. The

difference in moments between as built and monolithic construction are called the trapped moments. Once the bridge is constructed this way, the creep and shrinkage of concrete will gradually relieve the trapped moments thus a long time after the construction, the magnitude of trapped moments can be reduced to about 20-25% of the original trapped moments (Neville et al., 1983). This can be clearly seen with the detailed analysis carried out for Kylesku bridge in Edinburgh, United Kingdom (Nissen et al., 1985). When this effect is taken into account, there will be a bending moment envelope for the dead loads as well.

3. Determine the bending moments due to imposed and other loads and produce the bending moment envelope.
4. Select a suitable magnitude for the secondary moment at each support and alter the origin of the bending moment envelope. The magnitude of the secondary moments can be any value between 0% and about 70% of the as built dead load bending moment values since secondary moments are generally used to counteract the permanent loads.
5. Draw Magnel diagrams at each support section and check whether the feasible region is sufficient to have the tendons installed. If insufficient, increase the secondary moments further. If this cannot give a suitable feasible region, the bottom flange thickness selected may be too thin, thus would have to be increased.

The dimensions used for a prismatic prestressed concrete box girders can be optimised using this simple technique.

SELECTION OF THE CABLE PROFILE

Once the secondary moments are selected over the supports, the design engineer has to select a cable profile that will generate exactly the same as those assumed. This is not a trivial task and usually the design engineer may resort to iterative process. A design technique is presented that will almost eliminate the need for iterations. This is a combination of two design techniques available for the design of continuous bridges.

Line of Thrust Design

In line of thrust design, the secondary moments are treated as prestressing effects. This means that the secondary moments are unknown until the cable forces and profile are found. Therefore, secondary moments are not directly considered in the design. There are a number of stress conditions which need to be satisfied by the cable forces and eccentricities. If the horizontal component of the cable force is P, the area of concrete is A_c, the section modulus is Z, the permissible stress of concrete is compression is f_c, and the permissible stress of concrete in tension is f_t, then the stress conditions can all be presented in the following form using the bounds on the line of thrust, e_p.

$$f_c \leq -P/A_c - P\, e_p /Z + M/Z \leq f_t$$

This equation can be rearranged to give the bounds of e_p. Since there are two bounds at each cross section, it can form a line of thrust zone when the full length of the beam is considered. The designer has to find a profile which not only fits within the bounds of the line of thrust zone throughout the length of the beam, but is also concordant, which means that this profile does not generate any secondary moments. Then, this profile can be linearly transformed to fit within the bounds imposed by the concrete cover required for prestressing cables. The designer may have to resort to trial and error process if he finds that linearly transformed profile does not satisfy the physical bounds imposed by the concrete cover at all locations along the beam.

Actual Cable Profile Design

In actual cable profile design, the secondary moments (M_2) are assumed initially and treated as loads. Therefore, they appear in the eccentricity equation, which is now written in the terms of the actual cable profile.

$$-Z/A_c - f_c\, Z/P + (M+M_2)/P \geq e_s \geq -Z/A_c - f_t\, Z/P + (M+M_2)/P$$

Since there are two bounds at each cross section, this will result in a cable profile zone. A cable profile has to be found which not only satisfies the limits set on e_s, but also causes the assumed values of the secondary moments. In this method too, the designer has to resort to iteration when he finds that the cable profile selected does not generate the assumed secondary moments.

Proposed Design Method

The design method proposed to find the cable profile combines both these methods. The steps involved are as follows:

1. Find the cable profile zone using the Magnel diagram so that there is sufficient space to fit the prestressing cables at the critical sections, primarily over the supports and at the mid span sections. For this purpose, a set of prestressing forces also have to be selected, which can be done as explained in Section 5.4. Generally at critical sections, the bounds on the cable profile will not be governed by e_s, but by the physical limits imposed by the cover requirements.
2. Transform this cable profile with the actual limits (either e_s or physical limits due to cover) using the relationship which exist between the cable profile and the line of thrust, which is given by $e_s - e_p = M_2/P$. Since the secondary moments and prestressing forces have already been selected to determine the cross section, this transformation is possible.
3. Fit a concordant profile into the line of thrust zone, which means this profile does not generate any secondary moments. For the existence of a concordant profile, certain conditions should be satisfied by the upper and lower bounds of the line of thrust zone as explained in Section 5.4. The concordant profile can easily be found by fitting a bending moment diagram that correspond to some notional loads applied to the box

girder beam, since any bending moment diagram representing zero deflections at supports can be scaled down to form a concordant profile.

4. Transform the concordant profile back to the cable profile zone by using the assumed secondary moments and the prestressing forces selected. The resulting cable profile will generate secondary moments which are exactly the same as those assumed, thus eliminating the need for any iterations.

Selection of Cable Forces

The selection of the cable forces for a determinate prestressed concrete beam is straight forward task because the cable forces can be found directly from the Magnel diagram. However, this is not the case with continuous beams due to the existence of secondary moments. The cable force is additionally constrained because it has to ensure that the assumed secondary moment distribution is obtainable.

When the cable forces are selected, it is possible to obtain the bounds of cable profile zone and then transform those linearly to obtain the bounds of line of thrust zone. There is a unique criterion for the existence of a concordant profile within the line of thrust zone (Burgoyne, 1987 a). That is if a cable is fitted along the upper boundaries of the line of thrust zone, it should generate hogging secondary moments. If a cable is fitted along the lower boundaries of the line of thrust zone, it should generate sagging secondary moments. Then only a profile can exist that will generate zero secondary moments, thus concordant. Therefore, the designer can determine whether the magnitude of prestressing forces selected are sufficient by determining the magnitude of secondary moments with profiles fitted along the upper and lower boundaries of the line of thrust zone before trying to fit a scaled down bending moment diagram. If a concordant profile is not available, the designer will never fit a bending moment diagram within the line of thrust zone.

The condition for the existence of a line of thrust (a concordant profile) can be presented as follows. When a cable profile and the prestressing forces are known, it is possible to find the secondary moments by using the generalised Clark Maxwell's theorem or principle of virtual work (Burgoyne, 1987a). The main advantage is that the analysis can be performed symbolically. The associated equations are given in Appendix A. If the cable is concordant, the reactant moment at each support is zero. It is both a necessary and sufficient condition for this that the terms in the right hand side of Eq. A.3 are all separately zero. Thus a cable profile is concordant if:

$$\int \frac{\beta_i RPdx}{EI} = 0 \qquad \text{for all } i.$$

It is shown by Jayasinghe (1992) that it is more effective to increase the prestressing forces over the support regions than increasing them in the span regions for the existence of a concordant profile when different cable forces are used over the supports and span regions. This is because, the function β is close to 1.0 over the supports whereas it is a much lower value in the span regions. Therefore, the following strategy can be adopted for the selection of prestressing forces.

1. Select a minimum prestressing force in the span regions so that there is sufficient feasible region to fit the prestressing cable. For this purpose, the Magnel diagram should be drawn using the moments resulting from loading and secondary moments, thus $M + M_2$.
2. Select a suitable force over the support regions, but this can be larger than the minimum required. Select the relevant cable profile and then transform it to form the line of thrust zone. Check whether a concordant profile exist within this zone by calculating the secondary moments over the supports with the cable fitted at upper and lower boundaries of the line of thrust zone. This is further explained in the design example. If not, increase the prestressing force over the support further. This check will ensure if there is any iteration in the design process, it is only localized.
3. Select a concordant profile using a set of notional loads. Even this can be automated as described by Burgoyne (1987b) and Jayasinghe (1992).

USE OF GRILLAGE METHOD FOR ANALYSIS OF BOX GIRDERS

Grillage analogy is extensively used for the analysis of bridges due to reasons such as:

a. the grillage analogy can be used even in cases where the bridge exhibits complicated features like heavy skew, edge stiffening, deep haunches over supports, etc.
b. the representation of a bridge as a grillage ideally suited to carryout the necessary calculations associated with analysis and design on a digital computer,
c. the grillage representation is conducive to give the designer a feel for the structural behaviour of the bridge and the manner in which bridge loading is distributed and eventually taken to the supports.

In cellular bridges, longitudinal grillage beams are usually placed coincident with webs of the actual structure. The transverse medium, which consists of both the top and bottom flanges, is represented by equally spaced transverse grillage beams. The properties of the longitudinal beams are calculated by considering I sections as shown in Figure 2.

The structural action of the transverse medium of a cellular structure can be explained using Figure 3a, which shows the transverse slice of the structure being subjected to a vertical load at one end and an equlibrating moment at the other. If the plane sections remain plane thus there is negligible shear deformation, the transverse slice will deform as shown in Figure 3.b. However in cellular structures without frequent transverse diaphragms, the flanges and webs do flex significantly about their individual axes and this causes the cross section to distort as shown in Figure 3.c. Plane sections do not remain plane and the flexibility of the slices increases. The increase in flexibility cannot be accounted for by reducing the flexural rigidity of the equivalent beam because the additional deflections of the slice respond to shear in the slice rather than moments.

In the grillage analogy, this deformations of the cells in transverse direction can be taken into account by assigning an equivalent shear area for the transverse beams. The equivalent area can be found by using the Equation 1 (Jaeger & Bakht, 1982).

$$A_y = \frac{12L_x}{P_y}\frac{E_c}{G_c} \times \left[\frac{I_3(12I_1I_2 + L_xI_1I_3 + L_xI_2I_3)}{4P_yI_1I_2 + 4P_yI_2I_3 + 12H^2I_1I_2 + P_y^2I_3^2}\right] \qquad \text{Eq. 1}$$

The following notations have been used:

L_x = distance between transverse grillage members
P_y = distance between webs
I_1 = second moment of area of top flange per unit length in longitudinal direction
I_2 = second moment of area of bottom flange per unit length in longitudinal direction
I_3 = second moment of area of a web per unit length in longitudinal direction
H = height of the section as shown in Figure 4.

The second moment of area of the transverse members, I_y, can be found by using Equation 2 where the notations are as given in Figure 4.

$$I_y = L_x(t_1h_1^2 + t_2h_2^2) \qquad \text{Eq. 2}$$

The torsional inertia in the longitudinal direction, J_x, can be found by using Equation 3 where the notations are given in Figure 5.a.

$$J_x = L_y\frac{A_1^2}{b\oint\frac{ds}{n_st}} \qquad \text{Eq. 3}$$

The torsional inertia in the transverse direction, J_y, can be found by using Equation 4 where the notations are given in Figure 5.b.

$$J_y = L_x\frac{2A_2^2}{L\oint\frac{ds}{n_st}} \qquad \text{Eq. 4}$$

DESIGN EXAMPLE

In order to illustrate the design method a complete design example has been presented. The box girder bridge consist of three spans of 30m, 40 m and 30 m. This bridge will carry a two lane highway of lane width 3.0 m, also having two cycle paths of width 1.5 m each on either side and two walkways of width 1.0 m on either side. Thus the total top flange width is 11.0 m. The webs are selected at a spacing of 5.5 m, thus the overhang is 2.75 m. The thickness of the top flange selected is 0.35 m in average. The thickness of the webs selected is 0.35 m. The depth of the bridge is selected as 2.0 m thus giving a rather shallow bridge with span/depth ratio of 20 for the mid span. The average thickness of the bottom flange is selected as 0.175 m. It is also assumed that the large diameter cable used will necessitate the centroid of the cable to be at least 150 mm away from the top and bottom fibres. A cross section of the selected section is given in Figure 6.

This bridge is analysed for HA loading and 30 units of HB loading as given in BS 5400/2 using the grillage analogy. The grillage consists of two longitudinal beams, each representing a web and the two flanges connected to it. The transverse beams are arranged at 5.0 m spacing. The diaphragms over the supports are ignored since those are likely to have large openings for service runs and inspection purposes. The section properties used for each grillage member for the analysis are given in Table 3.

Table 2: Section Properties Used for the Analysis

	Longitudinal	transvers
Area (m^2)	2.922	0.0052
Second moment of area (m^4)	1.41	1.754
Torsional inertia (m^4)	0.741	0.508

The concrete used for the analysis is grade 40 with elastic modulus of 30 kN/mm^2. The Poisson's ratio, ν, is considered as 0.15. The allowable compressive stress is taken as -14 N/mm^2 and the allowable tensile stress is taken as -2N/mm^2. The reasons for these values can be explained as follows. If the structure is designed as a class 1 structure, the allowable working stress is -16N/mm^2 for grade 40 concrete and the allowable tensile stress is 0.0 (BS 5400/4). Since there can be temperature induced stresses and other secondary stresses of magnitude of about 2 N/mm^2, the above values have been used so that once the final stresses are checked, no adjustment will be required for the prestressing forces or cable locations.

Table 4a: Moments Used for Structural Design

Chainage	0	5	10	15	20	25	30	35	40	45	50
M as-built	0	3069	4455	4085	1961	-1919	-7552	-757	3739	6718	7942
M_2	0	503	1006	1509	2012	2515	3020	2879	2738	2597	2456
M_{min}	0	3317	3827	2932	-787	-6854	-17006	-5467	793	4447	5751
M_{max}	0	7116	10734	10783	6537	-1275	-12656	-1625	7109	12630	14702

Table 4b: Moments Used for Structural Design.

Chainage	55	60	65	70	75	80	85	90	95	100
M as-built	7411	5125	1084	-4713	460	3878	5541	5449	3602	0
M_2	2315	2174	2033	1885	1570	1256	942	628	314	0
M_{min}	4704	1307	-4512	-15398	-6378	-403	3223	4499	3424	0
M_{max}	13323	8495	573	-6846	1150	8070	11415	11186	7380	0

Bridge is considered as constructed with span by span construction technique with 5.0 m cantilevering to the next span. The dead load bending moment envelope is obtained by considering that 80% of the trapped moments will be relieved by the creep of concrete. The superimposed dead load is calculated for an asphalt concrete filling of 150 mm thickness. The imposed load envelope calculated consisted of a sagging and a hogging bending moment at each section. The bending moment envelope used for the design is given in Tables 4a & 4b. The secondary moments are assumed as 40% of the as built bending moments at the supports. For clarity, the as-built bending moments are also given. The bending moment envelope for the determination of cable profile zone can be calculated by adding the secondary moments to the bending moments due to dead and live loads.

Since the design calculations are of repetitive nature, those are carried out using a spread sheet program as given in Data sheet 1. In this example, a constant prestressing force of 38,000 kN has been used. The prestressing force effective after losses is 0.7 x 38,000. The prestressing force has been adjusted until the secondary moments due to a cable placed at the upper boundary of the line of thrust zone will produce hogging secondary moments at each support. At the same time, the cable placed at the lower boundary of the line of thrust zone should produce sagging secondary moments. The secondary moments obtained for a cable force 0.7 x 38000 kN are given in Table 5.

Table 5: Secondary Moments When the Cable is at Upper and Lower Bounds of Line of Thrust

Location	M_2 at first internal support	M_2 at second internal support
Cable at upper bound of line of thrust	-3779	-1356
Cable at lower bound of line of thrust	3315	1236

These secondary moments are calculated as given in Appendix A. The right hand side of Equation A.3 is obtained by using the Simpson rule. The calculations performed are given in Data sheet 1. The secondary moments produced by the actual cable profile selected is also calculated as a check. Those are given in Table 6.

Table 6: Secondary Moments Assumed and Those Actually Occur

Location	first internal support	Second internal support
Assumed secondary moment (kNm)	3020	1885
Actual secondary moment (kNm)	3032	1897

The concordant profile that fits within the line of thrust zone is calculated in the following manner. First the boundaries of the line of thrust zone has been converted to a force-eccentricity zone where the eccentricity has been multiplied by the prestressing force selected. A set of notional loads have been selected that produces a bending moment diagram that fits within the force-eccentricity zone. The notional loads considered consist of uniformly distributed element loads of the magnitudes given in Table 7. Negative sign indicates that the loads are acting downwards.

Table 7: Notional Loads Applied to the Beam to Obtain the Line of Thrust.

Chainage	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50	50-55
Notional load (kN/m)	-120	-120	-120	-120	-120	220	220	-135	-135	-135	-105

Chainage	55-60	60-65	65-70	70-75	75-80	80-85	85-90	90-95	95-100
Notional load (kN/m)	-105	-105	170	170	-105	-105	-95	-95	-95

The concordant profile is obtained by dividing the bending moments due to notional loads by the prestressing force of 0.7 x 38000. This concordant profile is transferred back to the cable profile zone by using the relationship, $e_s = e_p + M_2/(R.P)$, where R.P is the effective prestressing force. The actual stresses resulting from the cable profile selected also can be calculated. In this case, the stresses will satisfy the stress limits since the cable is within the boundaries of the cable profile.

CONCLUSIONS

A design method, that guides the designer as a logically evolving process has been presented. The main advantage of this design method is that the designer has the opportunity to select the section dimensions of the box section by considering the governing criterion for each component such as top flange, webs and bottom flange. It is shown that the secondary moments can be used to minimise the thickness of the bottom flange to minimum constructable thickness. This helps the designer to achieve the minimum cross section for the prestressed concrete section, thus allowing him to develop a solution quite close to the optimum that can be achieved. It should be noted that with this technique, the secondary moments have already been assumed at the end of the selection of section dimensions.

It is shown that there is a unique criterion to satisfy the existence of a concordant profile within the line of thrust zone. This condition can be used by the designer to check the existence of a concordant profile within the line of thrust zone that is obtained corresponding to the prestressing forces selected. This helps the designer to eliminate any iterations involved by carrying out this simple check and to alter the prestressing force if the need arises. The application of this technique is illustrated using a design example.

Since the secondary moments have already been assumed, it is necessary to ensure that the cable profile selected will generate those assumed moments. The steps that should be followed to fulfil this objective is also presented.

With this straight forward design technique, the design process has been considerably simplified and also it offers an engineer without much experience in the design of continuous bridges to obtain a close to optimum solution.

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Appendix A:

It is possible to determine the secondary moments in a beam once the cable profile and the cable forces are known by using the generalised Clark Maxwell's theorem which states that when two force systems act on a linear elastic structure, the work done by the forces of first system on the displacements of second system is equal to the work done by the forces of the second system on the displacements of the first system ($\Sigma P_i \Delta_{ii} = \Sigma P_{ii} \Delta_I$).

The first system consist of a set of fictitious moment system and associated reactions as shown in Figure A.1 with one system at each internal support. The reactions R'_{ik} are unknown, but need not be determined since they do not appear in the equations. The second system consist of the cable forces, corresponding secondary moments and associated reactions at the internal supports. Due to cable forces and secondary moments, there is a curvature at each section given by $(\Sigma(\beta M_2)_j - RPe)/EI$. The support reactions resulting from cable forces are R_{jk}, but those need not be calculated since they appear at the supports where the displacements are zero in the first system. The application of the generalised Clark Maxwell's theorem results in Equation A.1from which $(M'_2)_i$ can be cancelled from the equation without loosing the generality, thus resulting in Equation A.2.

$$\int \beta_i (M_2^{'})_i (\sum_j \beta_i (M_2)_j - Pe) \frac{1}{EI} dx = 0 \qquad \text{for } i = 2, 3, \ldots\ldots, n\text{-}1 \qquad \text{Eq. A.1}$$

$$\sum_j (M_2)_j \int \frac{\beta_i \beta_j}{EI} dx = \int \frac{\beta_i Pe}{EI} dx = \int \frac{\beta_i Pe}{EI} dx \quad \text{for } i = 2, 3, \ldots\ldots, n\text{-}1 \qquad \text{Eq. A.2}$$

The equations form a set of linear equations consisting of unknown secondary moments $(M_2)_j$. The cable profile, e, appears on the right hand side of the equations, and the coefficients of the $(M_2)_j$ terms are integrals of products of the β functions, many of which are zero since β_i is non zero only on either side of the support under consideration. The integration $\int \beta_i \beta_j dx$ results in simple expressions as indicated in the left hand side of the matrix notation given in equation A.3. The integral $\int \beta_i RP.e.dx/EI$ can be determined using numerical integration, preferably using the Simpson's rule, since the cable profile is known as a set of eccentricities with parabolic segments in between. In Data sheet 1, the values of β factors and the Simpson's rule coefficients (s) have been indicated. For each of the β coefficients, a row of β.RP e_s.s, have been prepared and then summed to give the totals shown at the end of the row. These values should be multiplied by h/3 as generally done in Simpson's rule summations where h is the interval considered for the cable eccentricities. The matrix can now be inverted to obtain the unknown secondary moments.

$$\frac{1}{6}\begin{bmatrix} 2(L_1+L_2) & L_2 & 0 & - \\ L_2 & 2(L_2+L_3) & L_3 & - \\ 0 & L_3 & 2(L_3+L_4) & - \\ - & - & - & - \end{bmatrix} \begin{bmatrix} (M_2)_2 \\ M_2)_3 \\ M_2)_4 \\ - \end{bmatrix} = \begin{bmatrix} \int \frac{\beta_2 RPe}{EI} \\ \int \frac{\beta_3 RPe}{EI} \\ \int \frac{\beta_4 RPe}{EI} \\ - \end{bmatrix} \qquad \text{Eq. A.3}$$

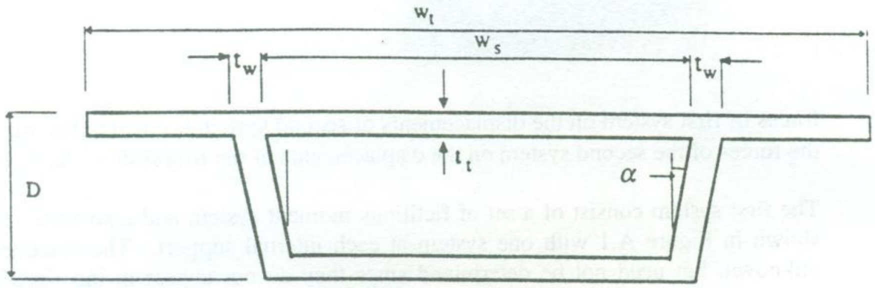

Figure 1: Idealization of a box girder for design

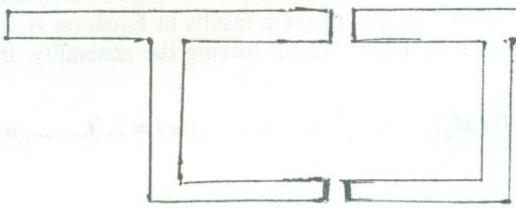

Figure 2: Box girder divided into I-girders

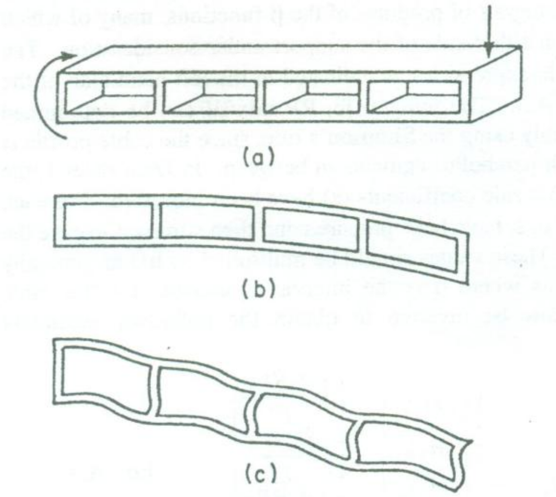

Figure 3: Transverse deformation of a cellular structure

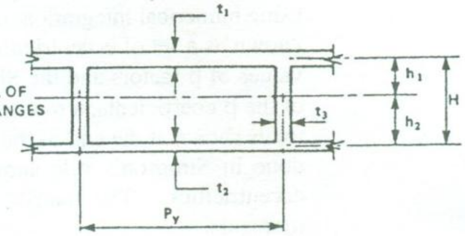

Figure 4: Partial cross-section of a cellular structure

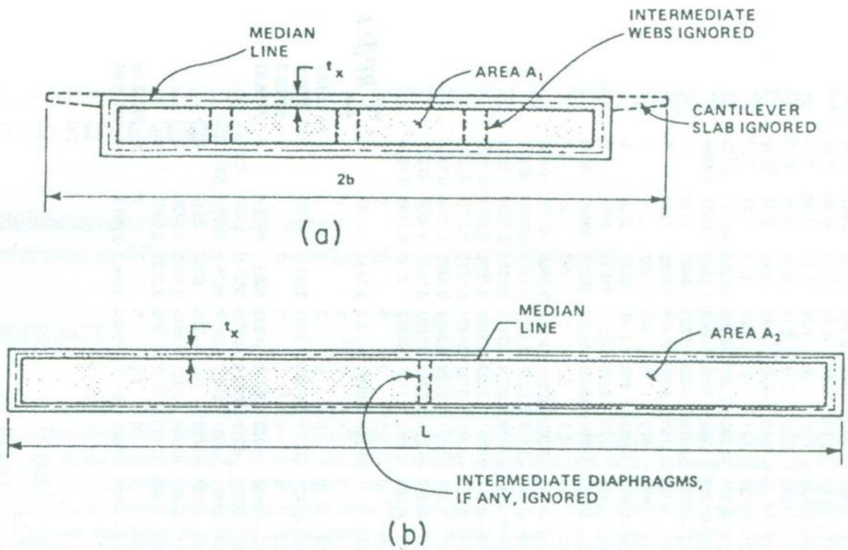

Figure 5: Areas considered in the calculation of torsional properties: (a) Cross section, (b) Longitudinal section

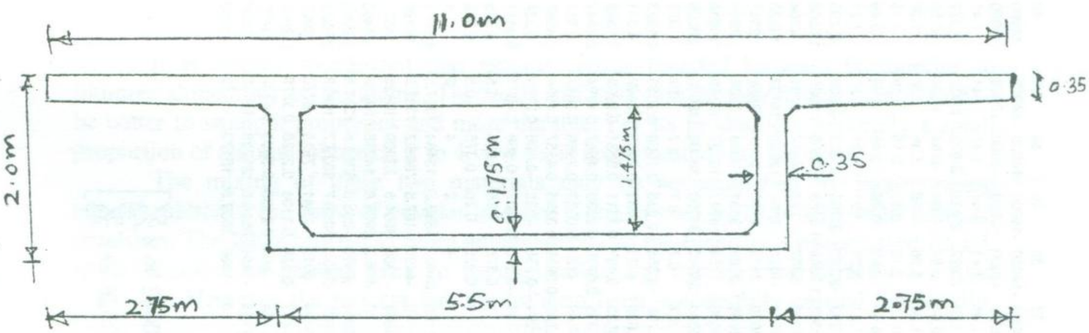

Figure 6: Section used for design

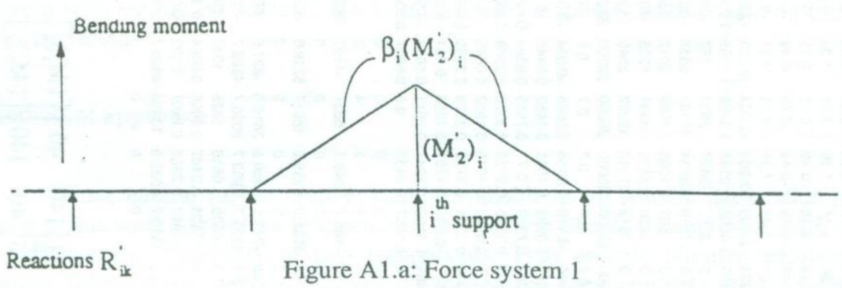

Figure A1.a: Force system 1

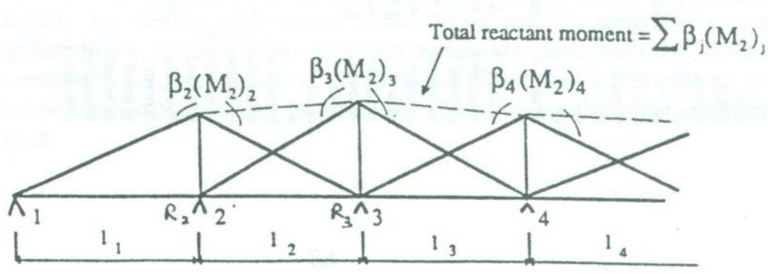

Figure A1.b: Reactant moment at the j^{th} support is $(M_2)_j$: Force system 2

chainage	0	5	10	15	20	25	30	35	40	45	50
fcw	-14000	-14000	-14000	-14000	-14000	-14000	-14000	-14000	-14000	-14000	-14000
ftw	-2000	-2000	-2000	-2000	-2000	-2000	-2000	-2000	-2000	-2000	-2000
Ic	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41
Ac	2.922	2.922	2.922	2.922	2.922	2.922	2.922	2.922	2.922	2.922	2.922
C1	-0.64	-0.64	-0.64	-0.64	-0.64	-0.64	-0.64	-0.64	-0.64	-0.64	-0.64
C2	1.36	1.36	1.36	1.36	1.36	1.36	1.36	1.36	1.36	1.36	1.36
cover	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
C1MIN	-0.49	-0.49	-0.49	-0.49	-0.49	-0.49	-0.49	-0.49	-0.49	-0.49	-0.49
C2MAX	1.21	1.21	1.21	1.21	1.21	1.21	1.21	1.21	1.21	1.21	1.21
z1	-2.203	-2.203	-2.203	-2.203	-2.203	-2.203	-2.203	-2.203	-2.203	-2.203	-2.203
z2	1.0368	1.0368	1.0368	1.0368	1.0368	1.0368	1.0368	1.0368	1.0368	1.0368	1.0368
Ma(actu)	0	3317	3827	2932	-787	-6854	-17006	-5467	793	4447	5751
Mb(actu)	0	7116	10734	10783	6537	-1275	-12656	-1625	7109	12630	14702
M2	0	503	1006	1509	2012	2515	3020	2879	2738	2597	2456
Ma	0	3820	4833	4441	1225	-4339	-13986	-2588	3531	7044	8207
Mb	0	7619	11740	12292	8549	1240	-9636	1254	9847	15227	17158
Ptrial	38000	38000	38000	38000	38000	38000	38000	38000	38000	38000	38000
R	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7
es upper 1	-0.406	-0.119	0.0358	0.0565	-0.084	-0.359	-0.768	-0.358	-0.035	0.1669	0.2395
es upper 2	-0.277	0.0096	0.1645	0.1852	0.0445	-0.23	-0.639	-0.23	0.0933	0.2956	0.3682
es lower 1	0.5883	0.7319	0.77	0.7553	0.6344	0.4252	0.0625	0.491	0.7211	0.8531	0.8969
es lower 2	0.1909	0.3345	0.3725	0.3578	0.2369	0.0277	-0.335	0.0936	0.3236	0.4557	0.4994
es upper	-0.277	0.0096	0.1645	0.1852	0.0445	-0.23	-0.49	-0.23	0.0933	0.2956	0.3682
es lower	0.1909	0.3345	0.3725	0.3578	0.2369	0.0277	-0.335	0.0936	0.3236	0.4557	0.4994
ep upper	-0.277	-0.009	0.1267	0.1285	-0.031	-0.325	-0.604	-0.338	-0.01	0.198	0.2758
ep lower	0.1909	0.3156	0.3347	0.3011	0,1613	-0.067	-0.448	-0.015	0.2207	0.358	0.4071
beta1	0	0.1667	0.3333	0.5	0.6667	0.8333	1	0.875	0.75	0.625	0.5
beta2	0	0	0	0	0	0	0	0.125	0.25	0.375	0.5
s	1	4	2	4	2	4	2	4	2	4	2
RPbeta1ep u	0	-165.7	2246.1	6837	-1103	-28798	-32108	-31463	-383.3	13164	7337.5
RPbeta2ep u	0	0	0	0	0	0	0	-4495	-127.8	7898.2	7337.5
RPbeta1ep lo	0	5595.9	5935.2	16017	5719.6	-5924	-23859	-1366	8804.5	23809	10828
RPbeta2ep lo	0	0	0	0	0	0	0	-195.2	2934.8	14286	10828
RPep upp	-7365	-248.5	3369.5	3418.5	-827.5	-8640	-16054	-8990	-255.5	5265.5	7337.5
RPep low	5076.7	8393.7	8903.7	8008.7	4289.7	-1777	-11929	-390.3	5869.7	9523.7	10828
concord	0	4808	6616	5425	1233	-5957	-11899	-5010	2941	7518	8720
epactual	0	0.1808	0.2487	0.2039	0.0464	-0.224	-0.447	-0.188	0.1106	0.2826	0.3278
es actual	0	0.1997	0.2865	0.2607	0.122	-0.129	-0.334	-0.08	0.2135	0.3803	0.4202
RPbeta1es	0	3540.7	5080.8	13868	4326.7	-11473	-17758	-7459	8518.5	25288	11176
RPbeta2es	0	0	0	0	0	0	0	-1066	2839.5	15173	11176

chainage	55	60	65	70	75	80	85	90	95	100	
fcw	-14000	-14000	-14000	-14000	-14000	-14000	-14000	-14000	-14000	-14000	
ftw	-2000	-2000	-2000	-2000	-2000	-2000	-2000	-2000	-2000	-2000	
Ic	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41	
Ac	2.922	2.922	2.922	2.922	2.922	2.922	2.922	2.922	2.922	2.922	
C1	-0.64	-0.64	-0.64	-0.64	-0.64	-0.64	-0.64	-0.64	-0.64	-0.64	
C2	1.36	1.36	1.36	1.36	1.36	1.36	1.36	1.36	1.36	1.36	
cover	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	
C1MIN	-0.49	-0.49	-0.49	-0.49	-0.49	-0.49	-0.49	-0.49	-0.49	-0.49	
C2MAX	1.21	1.21	1.21	1.21	1.21	1.21	1.21	1.21	1.21	1.21	
z1	-2.203	-2.203	-2.203	-2.203	-2.203	-2.203	-2.203	-2.203	-2.203	-2.203	
z2	1.0368	1.0368	1.0368	1.0368	1.0368	1.0368	1.0368	1.0368	1.0368	1.0368	
Ma(actu)	4704	1307	-4512	-15398	-6378	-403	3223	4499	3424	0	
Mb(actu)	13323	8495	573	-6846	1150	8070	11415	11186	7380	0	
M2	2315	2174	2033	1885	1570	1256	942	628	314	0	
Ma	7019	3481	-2479	-13513	-4808	853	4165	5127	3738	0	
Mb	15638	10669	2606	-4961	2720	9326	12357	11814	7694	0	
Ptrial	38000	38000	38000	38000	38000	38000	38000	38000	38000	38000	
R	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	
es upper 1	0.1823	-0.004	-0.308	-0.592	-0.303	-0.055	0.059	0.0386	-0.116	-0.406	
es upper 2	0.311	0.1242	-0.179	-0.463	-0.175	0.0737	0.1877	0.1673	0.0124	-0.277	
es lower 1	0.8522	0.7192	0.4951	0.0803	0.4076	0.6204	0.7449	0.7811	0.7289	0.5883	
es lower 2	0.4547	0.3217	0.0977	-0.317	0.0101	0.2229	0.3474	0.3836	0.3314	0.1909	
es upper	0.311	0.1242	-0.179	-0.463	-0.175	0.0737	0.1877	0.1673	0.0124	-0.277	
es lower	0.4547	0.3217	0.0977	-0.317	0.0101	0.2229	0.3474	0.3836	0.3314	0.1909	
ep upper	0.224	0.0425	-0.255	-0.534	-0.234	0.0265	0.1523	0.1437	0.0006	-0.277	
ep lower	0.3677	0.24	0.0212	-0.388	-0.049	0.1757	0.312	0.36	0.3196	0.1909	
beta1	0.375	0.25	0.125	0	0	0	0	0	0	0	
beta2	0.625	0.75	0.875	1	0.8333	0.6667	0.5	0.3333	0.167	0	$\sum RP\beta e.s$
s	4	2	4	2	4	2	4	2	4	1	
RPbeta1ep u	8937.7	565.25	-3396	0	0	0	0	0	0	0	-58330.2
RPbeta2ep u	14896	1695.7	-23770	-28421	-20714	940.66	8101	2547.4	10.35	0	-34100.9
RPbeta1ep lo	14671	3191.8	282.33	0	0	0	0	0	0	0	63705.3
RPbeta2ep lo	24452	9575.5	1976.3	-20643	-4338	6231.6	16599	6383.1	5678.4	0	73768.63
RPep upp	5958.5	1130.5	-6792	-14211	-6215	705.49	4050.5	3821.5	15.493	-7365	
RPep low	9780.7	6383.7	564.67	-10321	-1301	4673.7	8299.7	9575.7	8500.7	5076.7	
concord	6922	2499	-4548	-10784	-5424	747	4295	5280	3827	0	
epactual	0.2602	0.0939	-0.171	-0.405	-0.204	0.0281	0.1615	0.1985	0.1439	0	
es actual	0.3473	0.1757	-0.095	-0.335	-0.145	0.0753	0.1969	0.2221	0.1557	0	
RPbeta1es	13856	2336.5	-1258	0	0	0	0	0	0	0	50042.96
RPbeta2es	23093	7009.5	-8803	-17798	-12846	2670.7	10474	3938.3	2766.2	0	38626.99

$$\frac{1}{6}\begin{bmatrix}140 & 40\\ 40 & 140\end{bmatrix}\begin{bmatrix}(M_2)_2\\ (M_2)_2\end{bmatrix} = \begin{bmatrix}50042.96\times \frac{5}{3}\\ 38626.99\times \frac{5}{3}\end{bmatrix} \qquad \begin{bmatrix}(M_2)_2\\ (M_2)_2\end{bmatrix} = \begin{bmatrix}3032\\ 1897\end{bmatrix}$$

DATA SHEET 1