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Appendix 1

PARAMETERS OF THE TWO AREA POWER SYSTEM INVESTIGATED

(a) Parameters of the thermal system

Nominal frequency	= 50 Hz
Rated power of each area P_{r1}, P_{r2}	= 2000 MW
Nominal load of each area	= 1000 MW
Tie line synchronising coefficient T_{12}	= 0.25 pu
Equivalent load frequency droop R	= 2.0 Hz/puMW
Steam governor-turbine data:	
Governor actuator time constant T_G	= 0.1 s
Turbine time constant T_T	= 0.5 s

(b) Parameters of the mixed hydro-thermal system

Percentage of regulating hydro units		= 20%
Percentage of regulating thermal units		= 80%
Hydro governor-turbine data:		
Governor actuator time constants:	T_1	= 40 s
	T_2	= 0.513 s
	T_R	= 5.0 s
Penstock time constant	T_W	= 1.0 s

Rest of the data are as in (a) above

Appendix 2

IDENTIFICATION OF THE PARAMETERS OF THE MODEL USING LEAST SQUARES ESTIMATION

The least squares estimation for identification of model parameters will be discussed and a recursive least squares estimation for on line parameter estimation will be derived.

A2.1 Least Squares Estimation

Consider the system model given by equation (A2.1)

$$y(t) = q^{-k} \frac{B}{A} u(t) \tag{A2.1}$$

where,

$$A = 1 + a_1 q^{-1} + \ldots\ldots\ldots + a_n q^{-n}$$

and

$$B = b_0 + b_1 q^{-1} + \ldots\ldots\ldots + b_m q^{-m}$$

The coefficients of A and B are the unknown parameters to be estimated. Equation (A2.1) can be expressed in difference equation form as:

$$y(t) = -a_1 y(t-1) - a_2 y(t-2) \ldots\ldots\ldots - a_n y(t-n) + b_0 u(t-k) + b_1 u(t-k-1) + \ldots\ldots + b_m u(t-k-m) \tag{A2.2}$$

where,

$y(t)$, $y(t-1)$, ...$y(t-n)$, $u(t-k)$, $u(t-k-1)$......$u(t-k-m)$

are known output and input data.

and a_1, a_2, ... a_n, b_0, b_1, ...b_m are unknown model parameters to be estimated.

Let the number of unknown parameters to be N

Equation (A2.2) can be written for N sampling instances to obtain the following equations.

$$y(t-N+1) = -a_1\, y(t-N) - \ldots\ldots - a_n\, y(t-N+1-n) + b_0\, u(t-N+1-k) + \ldots\ldots\ldots + b_m u(t-N+1-k-m)$$

$$\vdots$$

$$y(t-1) = -a_1\, y(t-2) - \ldots\ldots - a_n\, y(t-1-n) + b_0\, u\,(t-1-k) + \ldots\ldots\ldots + b_m u(t-1-k-m)$$

$$y\,(t) = -a_1\, y(t-1) - \ldots\ldots - a_n\, y(t-n) + b_0\, u(t-k) + \ldots\ldots + b_m u(t-k-m) \qquad \text{(A2.3)}$$

The set of equations (A2.3) represent a set of N simultaneous linear equations with N unknowns, which can be solved to find the N unknown model parameters provided the N equations are independent.

Equation (A2.3) can be written in matrix form as:

$$\underline{Y} = X\,\underline{\theta} \qquad \text{(A2.4)}$$

where $\underline{Y}$ is a vector consisting of the output data

$$\text{i.e.} \quad Y = (y(t-N+1),\, y(t-N+2),\, \ldots\ldots\ldots\, y(t-1),\, y(t))^T \qquad \text{(A2.5)}$$

$\underline{\theta}$ is the unknown parameter vector given by

$$\underline{\theta} = (\,-a_1,\, -a_2,\, \ldots\ldots\, -a_n,\, b_0,\, b_1,\, \ldots\ldots b_m)^T$$

$$= (\,\theta_1,\, \theta_2,\, \ldots\ldots\ldots\, \theta_N)^T$$

and X is a (N,N) matrix consisting of past input-output data values:

$$X = \begin{bmatrix} y(t-N) \; \ldots\ldots\ldots & y(t-N+1-n) & u(t-n+1-k) & u(t-N+1-k-m) \\ \vdots & \vdots & \vdots & \vdots \\ y(t-2) & y(t-1-n) & u(t-1-k) & u(t-1-k-m) \\ y(t-1) & y(t-n) & u(t1k) & u(t-k-m) \end{bmatrix} \quad \text{(A2.6)}$$

Now, defining estimated parameters vector $\underline{\hat{\theta}}$ as:

$$\underline{\hat{\theta}} = (-\hat{a}_1 , \; -\hat{a}_2, \; \ldots\ldots \; -\hat{a}_n, \; \hat{b}_0, \; \hat{b}_1, \; \ldots\ldots \hat{b}_m)$$

$$= (\; \hat{\theta}_1, \; \hat{\theta}_2, \; \ldots\ldots\ldots \hat{\theta}_N)$$

where,

$\hat{\theta}_1$, $\hat{\theta}_2$, are the estimated values of

θ_1 and θ_2 respectively.

$\underline{Y}$ can be expressed as:

$$\underline{Y} = X \underline{\hat{\theta}} + \underline{e} \quad \text{(A2.7)}$$

where $\underline{e} = (e_1, e_2, \ldots\ldots e_n)^T$ is the estimation error vector

and e_1, e_2 ..... e_N are the estimation errors of

y(t-N+1), y(t-N+2), ... y(t) respectively.

The best estimate for $\underline{\theta}$ is obtained when the scalar product S given by $S = \underline{e}^T \underline{e}$ is minimum.

Since $\underline{e} = \underline{Y} - X \underline{\hat{\theta}}$, S can be expressed as:

$$S = [\underline{Y} - X\underline{\hat{\theta}}]^T [\underline{Y} - X\underline{\hat{\theta}}] \quad \text{(A2.8)}$$

The minimum of S is obtained when $\frac{ds}{d\underline{\hat{\theta}}}$ is zero.

i.e.

$$\frac{ds}{d\hat{\theta}} = [\underline{Y} - X\hat{\underline{\theta}}]^T X - X^T [\underline{Y} - X\hat{\underline{\theta}}]$$

$$= 2X^T [\underline{Y} - X\hat{\underline{\theta}}]$$

$$= 0$$

Hence, the best $\hat{\underline{\theta}}$ is given by

$$\underline{Y} - X\hat{\underline{\theta}} = 0$$

i.e. $$X\hat{\underline{\theta}} = \underline{Y}$$

$$X^T X\hat{\underline{\theta}} = X^T\underline{Y}$$

or $$\hat{\underline{\theta}} = [X^T X]^{-1} X^T \underline{Y} \qquad (A2.9)$$

Substitution of $\underline{Y} = X\underline{\theta}$ in equation (A2.9) gives:

$$\hat{\underline{\theta}} = [X^T X]^{-1} [X^T X] \underline{\theta} = \underline{\theta}$$

Therefore, $\hat{\underline{\theta}} = \underline{\theta}$ and correct estimates are obtained.

Also since $\underline{Y} = X\underline{\theta}$

and $\underline{Y} = X\hat{\theta} + \underline{e}$

$\underline{e}$ is given by $\underline{e} = X\underline{\theta} - X\hat{\underline{\theta}}$

Hence, for $\underline{\theta} = \hat{\theta}$, $\underline{e} = 0$.

A2.2 Recursive least squares algorithm

Consider the equations available at present time t when data samples have been collected over N+1 sample periods:

$$y(t-N) = -a_1\, y(t-N-1) \;\ldots\; -a_n\, y(t-N-n) + b_0\, u(t-N-k) + \ldots\ldots + b_m\, u(t-N-k-m)$$

$$\vdots$$

$$y(t-1) = -a_1\, y(t-2) \;\ldots\; -a_n\, y(t-1-n) + b_0\, u(t-1-k) + \ldots\ldots + b_m\, u(t-1-k-m)$$

$$y(t) = -a_1\, y(t-1) \;\ldots\; -a_n\, y(t-n) + b_0\, u(t-k) + \ldots\ldots + b_m\, u(t-k-m)$$

In matrix form this can be expressed as:

$$\underline{Y}_t = X_t\, \underline{\theta}_t \qquad \text{(A2.10)}$$

Equation (A2.10) can be written in partitioned form as:

$$\underline{Y}_t = \begin{bmatrix} \underline{Y}_{t-1} \\ \hline y(t) \end{bmatrix} = \begin{bmatrix} X_{t-1} \\ \hline \underline{x}_t^T \end{bmatrix} \underline{\theta}_t$$

where

$$\underline{Y}_{t-1} = [y(t-N), \ldots\ldots\; y(t-2),\, y(t-1)]^T$$

$$\underline{x}_t^T = [y(t-1), \ldots\ldots\; y(t-n),\, u(t-k), \ldots u(t-k-m)]$$

According to equation (A2.9), the best estimate $\hat{\underline{\theta}}_t$ is given by:

$$\hat{\underline{\theta}}_t = [X_t^T X_t]^{-1} [X_t]\, \underline{Y}_t$$

$$= \begin{bmatrix} [X_{t-1}^T \,\vdots\, \underline{x}_t] \begin{bmatrix} X_{t-1} \\ \hline \underline{x}_t^T \end{bmatrix} \end{bmatrix}^{-1} [X_{t-1}^T \,\vdots\, \underline{x}_t] \begin{bmatrix} \underline{Y}_{t-1} \\ \hline y(t) \end{bmatrix}$$

i.e. $$\hat{\underline{\theta}}_t = [X_{t-1}^T X_t + \underline{x}_t \underline{x}_t^T]^{-1} [X_{t-1}^T \underline{Y}_{t-1} + \underline{x}_t y(t)]$$ (A2.11)

Now, using the matrix identity given by:

$$[A^T A + \underline{a}\,\underline{a}^T]^{-1} = [A^T A]^{-1} - \frac{[A^T A]^{-1} \underline{a}\,\underline{a}^T [A^T A]^{-1}}{1 + \underline{a}^T [A^T A]^{-1} \underline{a}}$$

and letting

$$P_{t-1} = [X_{t-1}^T X_{t-1}]^{-1} \qquad (A2.12)$$

and $$d_{t-1} = 1 + \underline{x}_t^T P_{t-1} \underline{x}_t \qquad (A2.13)$$

Equation (A2.11) simplifies to:

$$\hat{\underline{\theta}}_t = [P_{t-1} - \frac{P_{t-1} \underline{x}_t \underline{x}_t^T P_{t-1}}{d_{t-1}}] [X_{t-1}^T \underline{Y}_{t-1} + \underline{x}_t y(t)]$$

$$= P_{t-1} X_{t-1}^T \underline{Y}_{t-1} + \frac{P_{t-1} \underline{x}_t}{d_{t-1}} [d_{t-1} y(t) - \underline{x}_t^T P_{t-1} (X_{t-1}^T \underline{Y}_{t-1} + \underline{x}_t y(t))]$$

$$= P_{t-1} X_{t-1}^T \underline{Y}_{t-1} + \frac{P_{t-1} \underline{x}_t}{d_{t-1}} [(1 + \underline{x}_t^T P_{t-1} \underline{x}_t) y(t) - \underline{x}_t^T P_{t-1} (X_{t-1}^T Y_{t-1} + \underline{x}_t y(t))$$

$$= P_{t-1} X_{t-1}^T \underline{Y}_{t-1} + \frac{P_{t-1} \underline{x}_t}{d_{t-1}} [y(t) - \underline{x}_t^T P_{t-1} X_{t-1}^T \underline{Y}_{t-1}]$$ (A2.14)

According to equation (A2.9) $\hat{\underline{\theta}}_{t-1}$ is given by:

$$\hat{\underline{\theta}}_{t-1} = [X_{t-1}^T X_{t-1}]^{-1} X_{t-1}^T \underline{Y}_{t-1}$$

$$= P_{t-1} X_{t-1}^T \underline{Y}_{t-1}$$

Therefore, equation (A2.14) can be expressed as:

$$\underline{\hat{\theta}}_t = \underline{\hat{\theta}}_{t-1} + \frac{P_{t-1}\,\underline{x}_t}{d_{t-1}} \left[y(t) - \underline{x}_t^T\, \underline{\hat{\theta}}_{t-1} \right] \qquad \text{(A2.15)}$$

Since $\underline{x}_t\, \underline{\hat{\theta}}_{t-1}$ is the predicted value of y(t) based on the old estimates $\underline{\hat{\theta}}_{t-1}$, the difference $y(t) - \underline{x}_t^T\, \underline{\hat{\theta}}_{t-1}$ represents the prediction error at time t.

$$\underline{G} \triangleq \frac{P_{t-1}\,\underline{x}_t}{d_{t-1}}$$ is a (n x 1) vector.

Hence equation (A2.15) can be expressed as

$$\underline{\hat{\theta}}_t = \underline{\hat{\theta}}_{t-1} + \underline{G}\;.\;(\text{Prediction error}) \qquad \text{(A2.16)}$$

Since $P_t = [X_t^T\; X_t]^{-1} = [X_{t-1}^T\; X_{t-1} + \underline{x}_t\; \underline{x}_t^T]^{-1}$

$$P_t = P_{t-1} - \frac{P_{t-1}\,\underline{x}_t\,\underline{x}_t^T\,P_{t-1}}{d_{t-1}}$$

i.e. $$P_t = P_{t-1} - \underline{G}\,.\,\underline{x}_t^T\,.\,P_{t-1} \qquad \text{(A2.17)}$$

Equations (A2.16) and (A2.17) provide a recursive algorithm for on line parameter estimation. The matrix inversion involved in the definition $P_{t-1} = [X_{t-1}^T\; X_t]^{-1}$ is avoided as P_t is evaluated in a recursive manner using equation (A2.17).

Appendix 3

Generalised Minimum Variance Control Law

A suitable control u(t), which minimises a general cost function will be derived.

Consider a model given by:

$$y(t) = q^{-k} \frac{B}{A} u(t) + \frac{C}{A} \xi(t) \qquad (A3.1)$$

and an auxiliary output ϕ defined as:

$$\phi(t+k) \triangleq P\, y(t+k) + Q\, u(t) - R\, w(t) \qquad (A3.2)$$

where, P, Q, R are weighting polynomials in

q^{-1} and w is the set point.

The control objective is to minimise:

$$I = E\{\phi^2(t+k)\} \qquad (A3.3)$$

i.e. $$I = E\{[P\, y(t+k) + Q\, u(t) - R\, w(t)]^2\} \qquad (A3.4)$$

y(t+k) in equation (A3.4) can be expressed in terms of u(t) and ξ (t+k) using equation (A3.1) and the cost function I can be rewritten as:

$$I = E\{[(\frac{PB}{A} + Q)\, u(t) - R\, w(t) + \frac{PC}{A} \xi(t+k)]^2\} \qquad (A3.5)$$

Now, expressing PC/A in the form:

$$\frac{PC}{A} = F + q^{-k} \frac{G}{A}$$

the last term in equation (A3.5), i.e. $\frac{PC}{A} \xi(t+k)$, can be expressed in terms of future values, and present and past values of the random disturbance.

i.e.

$$\frac{PC}{A}\xi(t+k) = F\xi(t+k) + \frac{G}{A}\xi(t) \tag{A3.6}$$

Since, F is of order k-1 the term $F\xi(t+k)$ involves only future values of the random disturbance.

Now, substituting for $\frac{PC}{A}\xi(t+k)$ in equation (A3.5):

$$I = E\{[(\frac{PB}{A} + Q)u(t) - R\,w(t) + F\xi(t+k) + \frac{G}{A}\xi(t)]^2\} \tag{A3.7}$$

The present and past values of the random disturbance can be calculated from the knowledge of A, B, C, k and the present and past values of u and y using equation (A3.1) as follows:

$$\xi(t) = \frac{A}{C}y(t) - q^{-k}\frac{B}{C}u(t)$$

then, the cost function I in equation (A3.7) modifies to:

$$I = E\{[(\frac{PB}{A} + Q)u(t) - R\,w(t) + F\xi(t) + \frac{G}{A}(\frac{A}{C}y(t) - q^{-k}\frac{B}{C}u(t))]^2\}$$

$$= E\{[(\frac{B}{C}(\frac{PC}{A} - q^{-k}\frac{G}{A}) + Q)u(t) + \frac{G}{C}y(t) - R\,w(t) + F\xi(t+k)]^2\}$$

$$= E\{[\frac{1}{C}(H\,u(t) + G\,y(t) + E\,w(t)) + F\xi(t+k)]^2\} \tag{A3.8}$$

where,

$$H = BF + QC \tag{A3.9}$$

and $$E = -RC \tag{A3.10}$$

Expanding equation (A3.8) results in:

$$I = E\{[\frac{1}{C}(Hu(t) + G\,y(t) + E\,w(t))]^2 + \frac{2F}{C}(Hu(t) + Gy(t) + Ew(t))\xi(t+k) + [F\xi(t+k)]^2\}$$

(A3.11)

Since the disturbance $\xi(t)$ is a random uncorrelated zero-mean sequence, the expected value of the middle term will be zero. This is because $F\xi(t+k)$ only involves future values of $\xi(t)$ which are uncorrelated with the present and past values of input, output and reference.

Hence,

$$I = E\{[\frac{1}{C}(Hu(t) + G\,y(t) + E\,w(t))]^2 + [F\,\xi(t+k)]^2\}$$

and for minimum I;

$$\frac{\partial I}{\partial u(t)} = 0$$

then,

$$\frac{\partial I}{\partial u(t)}\ E\{\frac{2H}{C}(H\,u(t) + G\,y(t) + E\,w(t))\} = 0$$

i.e., the control law which minimises the variance of $\phi(t+k)$ is given by:

$$\boxed{H\,u(t) + G\,y(t) + E\,w(t) = 0}$$

Appendix 4

AN ALGORITHM TO SOLVE $AH + q^{-1} BG = CT$

The solution algorithm is considered in the following three steps:

1) Transform the equation $AH + q^{-1} BG = CT$ into the form $M\underline{\mu} = \underline{\nu}$ by equating the coefficients of the equal powers of q^{-1} ;

2) Perform row operations to diagonalise the matrix M ;

3) Compute $\underline{\mu}$ by solving the equations obtained from (2) above.

Step 1 : form M and $\underline{\nu}$

Let the orders of the polynomials A, B, C and T to be n_A, n_B, n_C and n_T respectively.

Then, from equation (4.63) the orders of the polynomials G and H, i.e. n_G and n_H, are given by:

$$n_G = n_A - 1$$

$$n_H = n_B$$

(h_0 is fixed to unity)

Hence, the total number of unknowns is equal to $n_G + n_H + 1$; i.e. the total number of equations n_{EQ} is given by:

$$n_{EQ} = n_G + n_H + 1$$

From equation (4.64) the matrix M is given by:

$$M = \begin{bmatrix}
b_0 & & & & 1 & & & \\
b_1 & b_0 & & & a_1 & 1 & & \\
\cdot & b_1 & b_0 & & a_2 & a_1 & \cdot & \\
\cdot & \cdot & \cdot & & \cdot & a_2 & \cdot & \\
\cdot & \cdot & \cdot & & \cdot & \cdot & & 1 \\
b_{n_B} & & & b_0 & & \cdot & & a_1 \\
 & b_{n_B} & \cdot & \cdot & a_{n_A} & & & a_2 \\
 & & & \cdot & & a_{n_A} & \cdot & \cdot \\
 & & \cdot & \cdot & & & \cdot & \cdot \\
 & & & b_{n_B} & & & & a_{n_A}
\end{bmatrix}$$

M is formed as follows:

(a) set all the elements of M to zero.

$$m_{ij} = 0 \ , \ \text{for } j = 1, n_{EQ} \text{ and } i = 1, n_{EQ} \qquad (A4.1)$$

(b) form the first n_A columns.

i^{th} column is given by:

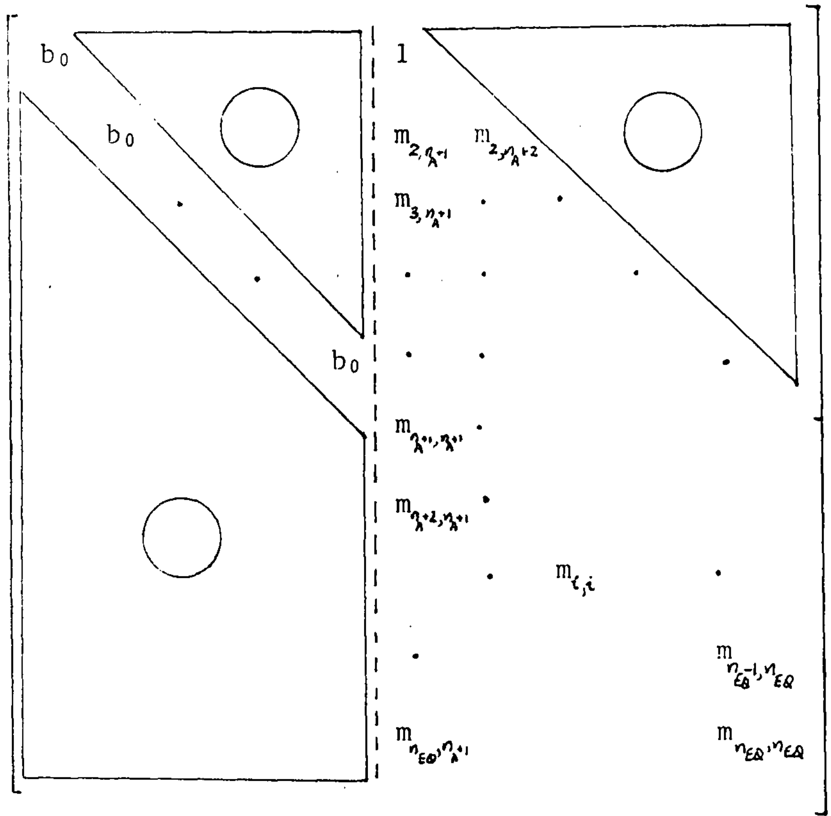
$$m_{i+j-1,i} = b_{j-1} \ , \quad j = 1, \ n_B + 1 \qquad (A4.2)$$

(c) form the rest of the columns.

In $n_A + i^{th}$ column, the first i-1 elements are zero, i^{th} element is 1.0 and i+1 th to $i+n^{th}$ elements are $a_1, a_2 \ldots\ldots\ldots a_{n_A}$ respectively.

Thus, $n_A + 1$ th column is given by:

$$m_{i,\ i+n_A} = 1.0 \qquad (A4.3)$$



(c) Row operations to set the elements below the diagonal of the columns $n_A + 1$ to n_{EQ} to zero.

number of elements below the diagonal elements $m_{i,i}$ is given by: $\ell = n_{EQ} - i$

let $R_k = m_{i+k,j} / m_{i,i}$ (A4.9)

Now, R_k time i^{th} row is subtracted from the $k+i^{th}$ row, for $k = 1, \ell$ and for $i = n_A + 1, n_{EQ} - 1$

Thus,

$$m_{i+k,j} = m_{i+k,j} - m_{i,j} R_k \quad \text{(A4.10)}$$

$$\nu_{i+k} = \nu_{i+k} - \nu_i R_k \quad \text{for } k=1, \ell \; ;$$

$$j = 1, n_{EQ} \; ; \; i = n_k+1, n_{EQ}-1 \quad \text{(A4.11)}$$

Step 3: Computation of G and H parameters

Compute μ_j for $j = n_{EQ}, n_{EQ} - 1, \ldots, 1$

and $m_{i+j,\ i+n_A} = -a_j$, for $j = 1, n_A$ (A4.4)

formed as follows:

i^{th} element of $\underline{\nu}$ is given by:

$$\nu_i = -a_i + c_i + t_i + \sum_{k=1}^{n_C} c_k\ t_{i-k} \quad \text{(A4.5)}$$

with $t_{i-k} = 0$ for $i-k > n_T$ and for $i-k < 0$

$t_{i-k} = 1$ for $i-k = 0$

$a_i = 0$ for $i > n_A$; $c_i = 0$ for $i > n_c$

Step 2: Diagonalise the matrix M

(a) Compute the ratio δ_i from:

$$\delta_i = b_i/b_{i-1}\ , \quad \text{for } i = 1, n_B \quad \text{(A4.6)}$$

(b) Row operations to set the elements below the diagonal of the first n_A columns to zero.

subtract δ_k times i^{th} row from j^{th} row for $i = 1, n_A$ and $j=i+1, i+n_B - 1$. where, $k=i-j$

thus,

$$m_{j,\ell} = m_{j,\ell} - m_{j-k,\ell}\ \delta_k \quad \text{(A4.7)}$$

and $$\underline{\nu}_j = \underline{\nu}_j - \underline{\nu}_{j-k} \cdot \delta_k \quad \text{(A4.8)}$$

Now the equation takes the form:

(a) for $j = n_{EQ}$

$$\nu_j = \nu_i / m_{jj} \qquad (A4.12)$$

(b) for $n_A < j < n_{EQ}$

$$\mu_j = [\nu_j - \sum_{k=1}^{n_{EQ}-1} m_{j,\, j+k} \; \mu_{j+k}] \; / \; m_{j,j} \qquad (A4.13)$$

(c) for $j < n_A$

$$\mu_j = [\nu_j - \sum_{k=n_A-j}^{n_{EQ}-j} m_{j,j+k} \cdot \mu_{j+k}] \; / \; m_{jj} \qquad (A4.14)$$