

ECONOMICS OF YIELD LINE ANALYSIS IN DESIGN OF CONCRETE SLABS

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ABSTRAC

Most practical structures are of three-dimensional nature, such as buildings, bridges, towers and tanks. These structures often require plate elements to do their intended function, or to make them stable. For reinforced concrete structures, these plate elements appear most often as horizontal reinforced concrete slabs. Efficiency achieved in analysis and design of these slabs is important, as these slabs normally are of high cost, because they contain large volumes of concrete and steel, in comparison to other elements such as beams, ties and columns

Further, various analytical methods available for slabs are considered to give different solutions as these slabs are highly indeterminate structures, and solutions depend on many variables such as shape, size, loadings and end conditions. However, range of local slabs widely used in Sri Lanka may be different from those popular in other countries. Hence there is a need for a local study to compare the performance of various analytical methods on the range of slabs widely used locally.

Analysis of reinforced concrete slabs is also considered more economical when plastic methods are used. But the greatest shortcoming of plastic methods of analysis is their inability to be amenable for computer applications. Further, few computer programs available commercially for plastic analysis are expensive and beyond the reach of local medium to small design offices. Thus the need, for a locally developed computer program based on yield line method, is important in the Sri Lankan context. This investigation is aimed to fulfill the former and the latter needs.

The investigation consisted a literature survey, a survey of reinforced concrete slabs obtained from structures designed in Sri Lanka, analytical and design study of these slabs, and work towards development of a computer program based on yield line theory' for the analysis of those identified slabs. A total of 70 slabs were included in



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