

Session 1

Numerical Analysis of Fluid Permeability in Compacted Sandstones

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Abstract

Fluid permeability in compacted sandstones were investigated numerically. The structural analysis of spherical particle bed which imitates compacted sand layer was conducted and pore region in the particle bed was extracted. The pore characteristics, such as effective porosity, tortuosity and specific surface area, were quantified by various numerical methods. From these pore characteristics, fluid permeability was calculated using the Kozeny-Carman equation. The obtained permeability agreed quantitatively with those measured from actual sandstones.

Keywords: Permeability; Pore characteristics; Kozeny-Carman equation; Sandstone

1. Introduction

Permeation flow in sedimentary layers is closely related to global resource and environmental issues such as radioactive waste disposal processes. An accurate evaluation of the permeability is important to tackle these problems. It is known that the permeability depends on the shape and size of the constituent particles. In this study, pore characteristics in compacted sandstones were examined numerically. The purpose of this study is to ascertain the applicability of the Kozeny-Carman equation [1], which is often used to predict the permeability of nearly spherical particle beds, to the actual sandstones consisting of deformed particles and its physical interpretation.

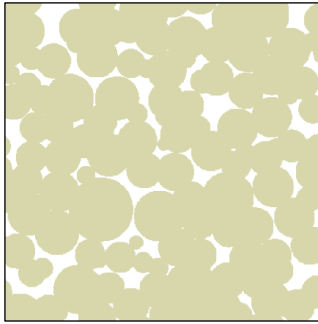
2. Numerical Methods

We simulated various particle beds composed of overlapped spheres imitating compacted sandstones. The polydispersed particles were randomly placed in the calculation domain and all particles were expanded by allowing overlaps. Figure 1 shows a comparison of the cross-sectional view of the spherical particle structure obtained by the numerical analysis (average particle diameter $d = 250$ mm with the variance $s_g = 0.2$) and that of an actual sandstone [2]. It is found that the numerical result is similar to the observation result regarding the shape and connectivity of pore networks.

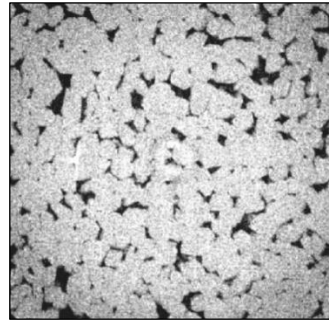
In the analysis, pore regions in the particle bed obtained numerically were extracted and the pore characteristics (effective porosity, tortuosity, and specific surface area) were calculated by various numerical methods. The obtained particle structure was subdivided into fine cells and categorized into various types. The effective porosity was calculated from the ratio of connected pore cells to the total number of cells. The specific surface area was calculated from the number of surface cells. The tortuosity was calculated by random walk analysis conducted in extracted pore cells. Then the permeability in the particle bed was calculated by substituting the pore characteristics into the Kozeny-Carman equation as follows [1].

$$k(f) = \frac{f^3}{2t^2(1-f)^2S_v^2} \quad (1)$$

where k is the permeability [m^2], f is the effective porosity, t is the tortuosity and S_v is the specific surface area [m^2/m^3].



(a) Numerical results



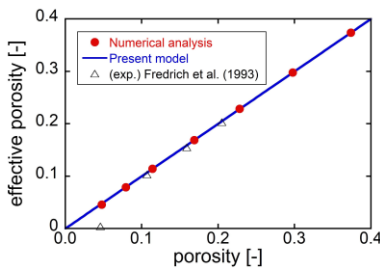
(b) Observation results [2]

Figure 1 Cross-section of spherical particle bed to experimental observation in sandstone.

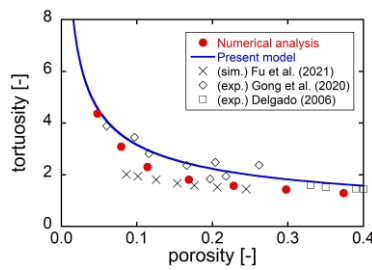
3. Results and Discussion

Figure 2 represents the numerical results of pore characteristics in particle bed with respective experimental results. Figure 2 (a) shows the results of effective porosity obtained from the analysis ($S_g = 0.2$) and the experimental results of sandstones [3]. Both results agree well over a wide range of porosity. Which indicates that the constitutive relation for the effective porosity in sandstone bed can be expressed by the following equation.

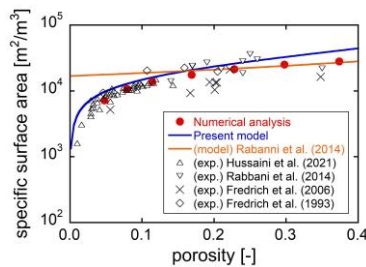
$$f_e(f) = f \tag{2}$$



(a) effective porosity



(b) tortuosity



(c) specific surface area

Figure 2 Numerical results of pore characteristics in particle bed with experimental results

The blue line in Figure 2 (a) represents the results obtained from Eq. (2). The model results are in quantitative agreement with the numerical and experimental results.

In the previous studies, the tortuosity was expressed by various functions of porosity [4]. One of the most common functions for tortuosity in spherical bed is a power law, for example, as follows.

$$\tau(f) = f^{-0.5} \quad (3)$$

Figure 2 (b) shows the results of tortuosity obtained from the present analysis with the experimental results of sandstones [5,6] and the simulation results in spherical beds [4]. The blue line in Fig.2 (b) indicates results obtained from Eq. (3). It can be seen that the model results agree with the numerical results and the experimental results.

Figure 2 (c) represents the comparison of specific surface area obtained from the numerical analysis and those of sandstones obtained experimentally [3,7-9]. The specific surface area ranges from $10^3 \sim 10^4 \text{ m}^2/\text{m}^3$ and exhibits a rapid decrease for $f < 0.1$. The orange line indicates the specific surface area model proposed by Rabbani et al., (2014) [9]. While the model results agree with experimental and numerical results at large porosity, it deviates from both results at low porosity. The reason for the inconsistency lies in the fact that the specific surface area should be zero at zero porosity.

We tried to develop a new geometric model which describes the specific surface area of sandstone for a wide range of the porosity. In the model, spheres in a hexagonal packing were expanded at an equal expansion rate and quantified the relation between the surface area and total volume of overlapped spheres. From the geometric calculation, the specific surface area is expressed by a function of porosity as follows.

$$S_v(f) = \frac{5\sqrt{\sqrt{2}p}}{d} \frac{\sqrt{f}}{1-f} \quad (4)$$

The blue line in Figure 3 (c) represents the model results of the specific surface area obtained from Eq. (4). The obtained specific surface area is in good agreement with the numerical and experimental results for a wide range of the porosity.

Figure 3 represents the experimental results of permeability of sandstones obtained from previous studies [2,10] and numerical results by the present analysis. The blue line indicates the permeability model as a function of the porosity, which is calculated from Eq.(1) with the constitutive relations of pore characteristics Eqs.(2), (3) and (4). The permeability model derived in the present study represents a close approximation to both the experimental and numerical results.

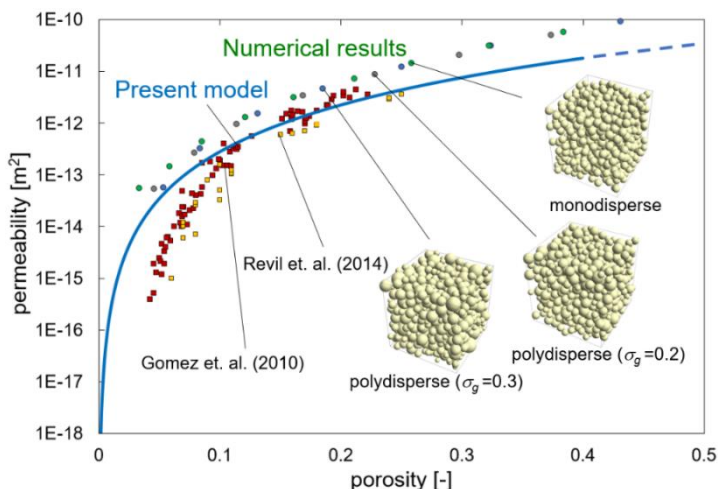


Figure 3 Experimental and numerical results and model of permeability of particle bed.

4. Conclusion

The structural analyses of spherical particle bed imitating sandstones were conducted in order to investigate pore characteristics in the bed. The effective porosity, tortuosity and specific surface area were calculated numerically from the extracted pore region. The obtained pore characteristics were compared with experimental results and were modeled as a function of porosity, respectively. The permeability was calculated by substituting these constitutive relations into the Kozeny-Carman equation. The resultant permeability was quantitatively consistent with the experimental results of actual sandstones. The present results imply that the original Kozeny-Carman equation is valid even in compacted sandstones since the increase of tortuosity cancels out the decrease of specific surface area at low porosity.

Acknowledgements

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