APPLICABILITY OF GEOMETRIC BROWNIAN MOTION AND GEOMETRIC FRACTIONAL BROWNIAN MOTION TO FORECAST SHARE PRICES OF TELECOMMUNICATION SERVICES SECTOR IN SRI LANKA

A.K.K.K. Athukorala

188852B

Thesis/Dissertation submitted in partial fulfillment of the requirements for the degree Master of Science

Department of Mathematics

University of Moratuwa

Sri Lanka

August 2022

DECLARATION

I declare that this is my own work and this thesis/dissertation does not incorporate without acknowledgement any material previously submitted for a Degree or Diploma in any other University or institute of higher learning and to the best of my knowledge and belief it does not contain any material previously published or written by another person except where the acknowledgement is made in the text.

Also, I hereby grant to University of Moratuwa the non-exclusive right to reproduce and distribute my thesis/dissertation, in whole or in part in print, electronic or other medium. I retain the right to use this content in whole or part in future works (such as articles or books).

Signature : Date :

The above candidate has carried out research for the Masters thesis/ dissertation under my supervision.

Signature : Date :

ABSTRACT

The Brownian motion is a Mathematical concept which European botanist Robert Brown introduced in 1827 to study the behaviour of molecules. The Brownian motion concept was transformed into many versions, and Geometric Brownian Motion (GBM) and Geometric Fractional Brownian Motion (GFBM) is the latest transformation of this concept. The GBM and GFBM are mathematical models used to forecast prices of stocks, commodities, etc. In this study, the GBM and GFBM were tested to estimate the share prices of telecommunication industry companies in Sri Lanka. The two sample companies were selected by representing 18% of the population of the telecommunication industry group. The five-year share prices were collected from sample companies: Sri Lanka Telecom PLC and Dialog Axiata PLC. The two models were implemented by estimating parameters such as the drift, the volatility, probability measurement and the time interval. In addition, the Hurst component was generated by a MATLAB program for GFBM. This study is concluded that GBM is the most accurate model for forecasting share prices of the telecommunication industry group with minimum mean absolute percentage error (MAPE).

Affectionately dedicated

to My Loving family members

for their enthusiastic encouragement,

which always helped me to achieve success

in my life.

ACKNOWLEDGEMENT

I would like to express my heartfelt gratitude and sincere thanks to my supervisor Mr. Rohana Dissanayake for giving me much of his precious time carefully supervising and guiding me throughout the study. In addition, I am very thankful for his lectures on Financial Mathematics which was very useful for understanding key concepts of this study. Secondly, I would like to thank the course coordinator, Dr. J.A.D. Miuran Dencil for his assistance in completing this research.

My warmest appreciation goes to my loving father and mother, Siripala Athukorala and Jayantha Amarasinghe, who are the giant partners in my success. This will be only a dream without my loving parents. I'm also very thankful to my two sisters Laxmi Athukorala and Dilani Athukorala and my two brothers-in-law Neluka Karannagoda and Chulaka Perera for standing with me every up and down in my life. Last but not least my sincere thank goes to my two little angels Thinaya Karannagoda and Enuki Perera for making me always happy.

TABLE OF CONTENT

Chapter 1 : Introduction

1.1. Background **1**

1.1.1. Stock Exchange 1

1.1.2. Sri Lanka Telecom PLC 3

1.1.3. Dialog Axiata PLC 3

1.1.4. Brownian Motion Concept 3

1.2. Objectives **4**

1.2.1. Primary objective 4

1.2.2. Specific objectives 4

1.2.3. Research question 4

1.3. Significance of the study **5**

1.4. Research outline **5**

2.1. Introduction **7**

2.2. Conceptual Framework **8**

2.3. Theoretical Literature **10**

Chapter 2 : Literature Review

2.3.1. Random Walk (Drunkard's Walk) 10

2.3.2. Brownian Motion (Wiener Process) 10

2.3.3. Generalized Random Walk 11

2.3.4. Stock Return 11

2.3.5. Ito's Lemma 12

2.3.6. Solution of Stochastic Differential Equation (SDE) 12

2.3.7. Drift & Volatility 13

2.3.8. Log Normal Distribution 14

2.3.9. The Hurst Component 14

2.3.10. Geometric Fractional Brownian Motion Model 15

2.3.11. Drift & Volatility with Hurst Component 15

2.4. Empirical Literature **16**

Chapter 3 : Methodology

3.1. Introduction **19**

3.2. Research Philosophy **20**

3.3. Research Approach 20
3.4. Methodological Choice 20
3.5. Research Strategy 21
3.6. Time Horizon 21
3.7. Data Collection & Analysis 21
3.7.1. Population 21
3.7.2. Sample 23
3.7.3. Sampling Method 24
3.7.4. Data Collection Method 25
3.7.5. Tools 25
3.8. Analytical Framework 25
3.8.1. Geometric Brownian Motion (GBM) Model 25
3.8.1.1. Calculation of Annual Volatility 26
3.8.1.2. Calculation of Annual Drift 27
3.8.1.3. Calculation of Probability Measurement 27
3.8.2. Geometric Fractional Brownian Motion (GFBM) Model 27
3.8.3.1. Calculation of Hurst Component 28
3.8.3.2. Calculation of Volatility with the Hurst Component 29
3.8.3.3. Calculation of Drift with the Hurst Component 29
3.8.3. Calculation of Mean Absolute Percentage Error (MAPE) 30
Chapter 4 : Analysis
4.1. Descriptive Statistical Analysis 31
4.1.1. Graphical representation of actual data 32
4.2. Geometric Brownian Motion (GBM) & Geometric Fractional Brownian Motion (GFBM) Model 34
4.2.1. Graphical representation of forecasted data 35
5.1. Discussion 38
5.2. Conclusion 39
5.3. Recommendations 40
5.4. Recommendations for Further Researches 40
Bibliography 41
Appendix A

LIST OF FIGURES

Figure 1.1	:	The industry group of telecommunication services	2
Figure 2.1	:	The summary of literature review	7
Figure 3.1	:	Saunder's research onion model	19
Figure 3.2	:	The structure of GICS	22
Figure 3.3	:	The hierarchy of communication services	22
Figure 3.4	:	Sampling methods	24
Figure 4.1	:	The time series plot of actual data – SLT	32
Figure 4.2	:	The time series plot of actual data – Dialog	33
Figure 4.3	:	Actual and forecasted share prices using GBM – SLT	36
Figure 4.4	:	Actual and forecasted share prices using GFBM – SLT	36
Figure 4.5	:	Actual and forecasted share prices using GBM – Dialog	37
Figure 4.6	:	Actual and forecasted share prices using GFBM – Dialog	37

LIST OF TABLES

Table 2.1	:	CSE industry groups	9
Table 3.1	:	No of share prices per year	25
Table 3.2	:	Interpretation of MAPE	30
Table 4.1	:	The summary of descriptive analysis	31
Table 4.2	:	Table of formulas	34
Table 4.3	:	Parameter estimation for GBM and GFBM	35
Table 4.4	:	Comparison of last data points of SLT	35
Table 4.5	:	Comparison of last data points of Dialog	35
Table 5.1	:	MAPE values	39

LIST OF APPENDICES

Appendix A	:	MATLAB program	43

LIST OF ABBREVIATIONS

GBM	:	Geometric Brownian Motion
GFBM	:	Geometric Fractional Brownian Motion
CSE	:	Colombo Stock Exchange
GICS	:	Global Industry Classification Standard
ASPI	:	All Share Price Index
SLT	:	Sri Lanka Telecom PLC
DIALOG	:	Dialog Axiata PLC
TRC	:	Telecommunication Regulatory Commission of Sri Lanka
St Dev	:	Standard Deviation
Var	:	Variance
MAPE	:	Mean Absolute Percentage Error