Mathematical Modelling of Mineral Processing Operations

by

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Abstract : During the last few decades, efforts have been made to describe the various unit operations involved in mineral processing by mathematical models. These models are of great importance in the design of new equipment and improving the efficiency of existing industrial plants. Also, the models have been used in the computer simulation and control of various processing operations which otherwise require. and depend on, highly experienced and skilled operators to ensure their profitability.

This paper briefly outlines the various models and/or the methods used in their development, in relation to a number of mineral processing unit operations such as screening. crushing/grinding and flotation.

1. Introduction

1.1. General

Computers have now become an integral part of any efficient and reliable operation. Whether it is purely in the trade and business sector or in the engineering sector, computers have taken over to do most of the tedious, routine and boring jobs, as well as most important decision making. reliably and efficiently.

In developed countries, like in all other areas, computers are being extensively used in the area of mineral processing, to ensure more efficient and profitable operations which otherwise demand the need for highly skilled and reliable operators for decision making. With the development of on-line measuring systems and understanding of the process behaviour, computers have been employed for the optimisation of performance of a given piece of equipment or a processing plant and also for automatic process control.[1] These techniques have proved to be capable of predicting the areas of inefficiency in a processing operation thus leading to the design of new equipment with improved performances.

However, in Sri Lanka, computer applications have not caught up with the mineral processing industry as yet. But with the increasing competition in the overseas markets, now we begin to feel the need for such applications to ensure the quality of our marketable products at lower processing costs.

The key to the use of computers for such applications is mathematical modelling. Mathematical modelling is to describe the working of a given piece of equipment or an operation as a whole, by mathematical relationships which could be easily fed to a computer.

The main aim of this paper is to give those who are involved in the mineral processing industry, the basic concepts that have been developed towards the mathematical modelling of mineral processing operations over the past few decades.

1.2. Mineral Processing

Mineral processing can broadly be divided into two areas,

viz. (*a*) Physical processing.
(*b*) Chemical processing.

Physical processing is generally the primary stage of processing which exploits the physical properties of the minerals involved such as size, shape, density, and surface properties etc. This stage is also known as the mineral dressing stage. The ore which has undergone the mineral dressing stage is then passed on to the chemical processing stage which usually produces the end product.

The kinetics of operation of chemical processing techniques such as leaching, ion-exchange, solvent extraction and subsequent operations such as electro-winning and electro-refining have been well understood and documented.

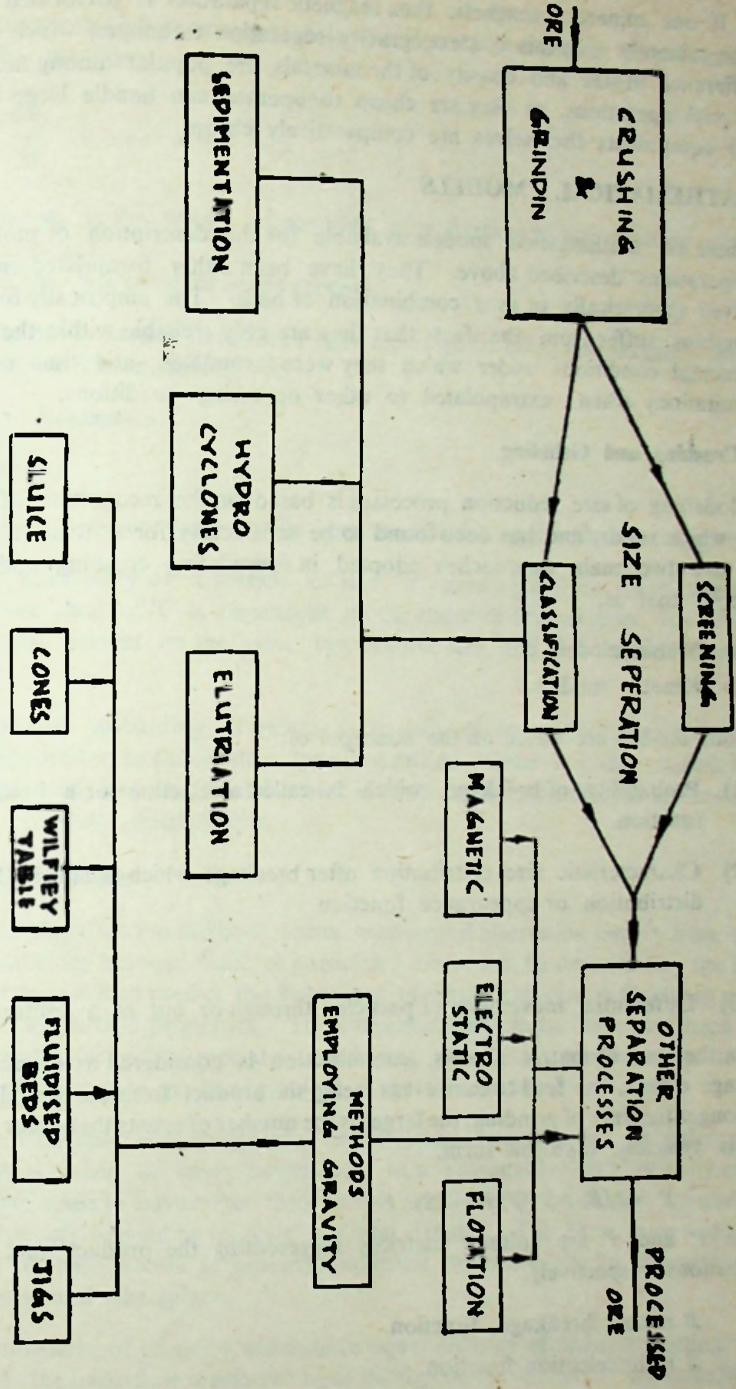
However, the understanding of physical processing operations is notyet complete. An attempt is made here to indicate very briefly, the basics of the approaches adopted for their descriptions.

1.3. Physical Processing of Minerals

The different techniques, generally employed in physical processing of a given ore, are shown schematically below.

In processing a given ore, crushing and grinding is of prime importance which liberates the valuable mineral from the gangue material. It is by far the most energy consuming and hence the most expensive unit operation and also the first stage of a processing operation. The ground material is then processed according to size, either by sieving (screening as the Americans call it) or by classification which uses the movement of solid particles in a fluid (usually water). This stage is important, as the efficiency of many of the subsequent processing equipment is low when their feed material is not uniform in size.

The classified ore which comprises of fully or partially liberated valuable mineral and the gangue is then subjected to one or more selected separation processes depending on the nature and the extents of the differences in the properties of the two minerals present.


ORE
CRUSHING & GRINDIN
SIZE SEPERATION
SCREENING
CLASSIFICATION
SEDIMENTATION
HYDRO CYCLONES
ELUTRIATION
OTHER SEPARATION PROCESSES
PROCESSED ORE
MAGNETIC
ELECTRO STATIC
FLOTATION
METHODS EMPLOYING GRAVITY
SLUICE
CONES
WILFIEY TABLE
FLUIDISED BEDS
JIGS

eg. If one mineral is magnetic then magnetic separation is performed to separate the minerals. In this context, gravity separation technique, which exploits the difference in size and density of the minerals are popular among most of the commercial operations, as they are cheap to operate, can handle large tonnages and the equipments themselves are comparatively cheap.

2. MATHEMATICAL MODELS

There are mathematical models available for the description of most of the unit operations described above. They have been either formulated empirically or derived theoretically or as a combination of both. The empirically formulated relationships suffer from the fact that they are only reliable within the range of experimental conditions under which they were formulated, and thus raises serious limitations when extrapolated to other operating conditions.

2.1. Crushing and Grinding

Modelling of size reduction processes is based on the recognition of physical events which occur, and has been found to be satisfactory for simulation purposes. There are two main approaches adopted in formulating crushing and grinding models,[2] that is,

(*a*) Matrix models

(*b*) Kinetic models.

Both models are based on the concepts of,

(1) Probability of breakage, which is called a selection or a breakage-rate function.

(2) Characteristic size distribution after breakage, which is called a breakage, distribution or appearance function.

and

(3) Differential movement of particles through or out of a continuous mill.

In the case of matrix models, comminution is considered as a succession of breakage events, the feed to each event being the product from the preceding event. The longer the time of grinding, the larger is the number of events that occur. Matrix models typically take the form,

$$P = (B \cdot S + I - S) . F$$

Where P and F are column matrices representing the product and feed size distributions respectively.

B is the breakage function.

S is the selection function.

and I is the identity matrix.

In kinetic models, comminution is considered a continuous process and generally take the form,

$$\frac{dw_i}{dt} = - k.w_i$$

Where w_i is the weight of particles of a particular size fraction present in the mill at time t
and k is the rate constant of the process.

These basic models have been later modified to take into account the effects such as classification, mixing etc.[2]

2.2. Size Separation

2.2.1. Sieving

Sieving operations have usually been modelled using a probabilistic approach.

The probability of a particle of size 'd' passing a given sieve aperture 'a' in a sieving time of 't' is dependent on the shape of the particles, the weight of the material present on the sieve, the relative size, and also the method of vibration.

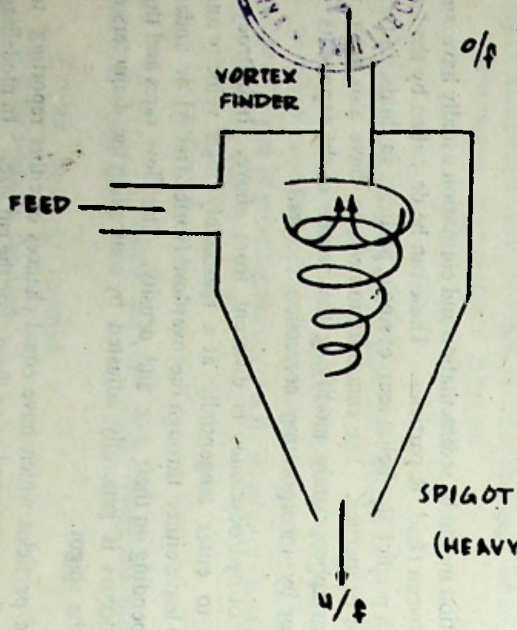
Thus, the probability of passing is usually evaluated by experiment under conditions similar to the proto - type operation. Once it is determined, it could be incorporated into the model which is then used to predict the under-size and over-size product distributions.

2.2.2. Classification

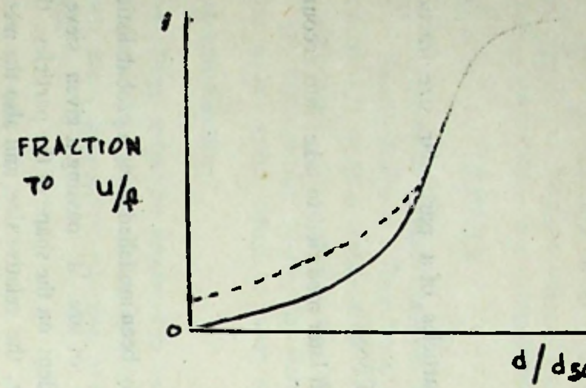
Of the classification methods, sedimentation and elutriation usually have smaller relative velocities between fluid & particles. These can be described by the laws of fluid mechanics which predict the behaviour of single particles in fluids, in terms of the fluid and particle properties. The concentration effects have also been taken into account by either appropriately modifying the values of the exponents in the derived equations or by incorporating correction factors.[3]

The behaviour of hydrocyclones is different from above. In hydrocyclones the slurry is made to enter tangentially at a reasonably high velocity and the particles are made to leave either through the overflow (vortex-finder) or underflow opening (spigot) depending on their size and density. The flow rates and the size at which the split occurs is generally adjusted by controlling the diameters of the vortex-finder and the spigot.

The diameter of particles which have equal chances of either reporting to the overflow or the underflow is referred to as the d_{50} or the cut size. In modelling the behaviour of hydrocylones a lot of effort has been put into the determination of their reduced efficiency curves which are considered to be characteristic of the machine.



SCHEMATIC DIAGRAM OF HYDROCYCLONE.



REDUCED EFFICIENCY CURVES

These curves have then been described by mathematical equations. The d_{50} size has been found to be a function of the vortex finder and spigot diameters, the inlet pressure and fraction of water reporting to the overflow and their relationaship to d_{50} has been evaluated empirically.[4]

2.3. Other separation processes

2.3.1. Gravity methods

Until recently the mathematical description of gravity separation techniques involving equipment such as tables, jigs, sluices, cone concentrators etc. has been limited to empirical relationships which relate the product grade and / or recovery to the operating variables.

Gravity techniques generally use the suspension of particles in a fluid and subjecting them to some form of shear in order to achieve a separation according to the size, density, shape of particles and also the fluid properties. Recent developments, however, show that these operations could be described by models which have been derived using the theories of fluid mechanics and mineral processing.[5,6] In many cases, the segregation of heavy minerals beneath the light gangue mineral has been shown to obey a first order law with rate constants which are functions of the materials used.

2.3.2. Flotation

Flotation is one of the most important separation processes which employ the principle of preferentially absorbing an air bubble to one mineral surface than to another. The surface characteristics of different minerals could be "conditioned" by the addition of reagents to achieve the required affinity for air. The mineral particles so conditioned are then allowed to cling on to a rising stream of air bubbles which take them to the surface, where they are skimmed off. This principle has been widely used in the mineral processing industry to separate the valuable mineral from the gangue material or to remove the last traces of impure material from a clean concentrate.

This process has been modelled using two techniques,[4] that is.

(*a*) by probability models

(*b*) by kinetic models.

Probability models take into account the probability of a particle being floated within a particular cell while the kinetic model assumes a first order rate law in determining the concentration of the suspension with time. Both models require the determination of the relevant constants by experimentation.

2.3.3. Magnetic and Electrostatic Methods

Separation processes which use magnetic or electrostatic properties have been modelled in the past mainly by empirically determined relationships. As the extents of the forces involved and the way in which they are applied to the mixture of particles differ from one machine to another, it has not been possible to derive a general law for such machines.

3. GENERAL COMMENTS

The above is only a brief attempt to indicate to the personnel involved in the mineral processing industry in Sri Lanka, the different techniques they could use in formulating models for their industries. The basic principles described above have been used in formulating computer models for a variety of unit operations at the Dept. of Mining & Mineral Engineering, University of Moratuwa and are on display at the present symposium.

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