

**COMPARISON OF EUROCODE 2 AND BS 8110  
RECOMMENDATIONS FOR THE DESIGN OF  
BENDING AND DEFLECTION IN SIMPLY  
SUPPORTED ONE WAY SLABS**

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Degree of Master Science in Structural Engineering Design

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NOVEMBER 2021

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Thesis submitted in partial fulfillment of the requirements for the degree of Master of  
Engineering

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November 2021

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Specially dedicated to my beloved family, teachers and friends...

## **Acknowledgements**

I would first like to express my sincere appreciation to the supervisors, Prof. W.P.S. Dias for his valuable advices and guidance given during this research study. I would like to express my gratitude to Dr.K.Baskan, the research coordinator, for the motivation and guidance he gave me to complete the thesis. Also, I thank the staff members of civil engineering department of University of Moratuwa for their kind assistances.

I wish to acknowledge my friends and colleagues for their feedback, cooperation and encouragement. Also, my most heartfelt appreciation goes to my family who give me invaluable support as always.

Finally, I am grateful to everyone who helped me in various ways to complete this research.

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## Abstract

Recently in Sri Lanka, design of the civil structures is adopted by Eurocodes superseded by the British code of practices. For reinforced concrete structures the Eurocode 2 will become of paramount importance to the design of the structural members. Bending and deflections are most important governing criteria in designs of slabs. Both standards for reinforced concrete design to check the deflection control by means of minimum member thickness requirement and a direct computation method. This research covers an analytical study that compared maximum span to depth ratios on deflection on different influence factors in simplified method and rigorous method. One-way slab on simply supported slab was considered to compare the parameter to compare in the research.

Further the deflections were compared with an experimental result produced by Gilbert (2004) with numerical calculation with respect to both code recommendation and its reliability was discussed. The influence factors on deflection and bending were analyzed in terms of sensitivity factors to understand the variation on results. The results indicated the EC2 predicts less area requirement for the flexure and a very thin slab can be used to control deflection if the applied moment not induced any crack in the element and the increased in tensile strength and Elasticity of concrete provide more sophisticated deflection control compared to the BS 8110.

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## LIST OF ABBREVIATIONS

Notation is commonly in accordance with Eurocode 2 and the principal List of Acronyms and Abbreviations are presented below.

The common system of subscripts such that the first subscript refers to the material, such as (c - concrete and s - steel), and the second subscript refers to the form of stress, such as (c – compression and t - tension).

<b>Abbreviation</b>	<b>Description</b>
$E$	Modulus of elasticity
$F$	Load (action)
$G$	Permanent load
$I$	Second moment of area
$M$	Moment (bending moment)
$Q$	Variable load
$a$	Deflection
$b$	Breadth (width)
$d$	Effective depth
$d'$	Depth to compression reinforcement
$h$	Overall depth of section in plan of bending
$i$	Radius of gyration
$k$	Coefficient
$l$	Length (Span)
$n$	Ultimate load per unite area
$1/r$	Curvature of a section/bending
$t$	Thickness
$x$	Neutral axis depth
$z$	Lever arm
$A_c$	Concrete cross-section area
$A_s$	Cross-section area of tension reinforcement
$A_s'$	Cross-section area of compression reinforcement
$A_{sreq}$	Cross-section area of tension reinforcement required at the ultimate limit state
$A_{s,prov}$	Cross-section area of tension reinforcement provided at ultimate limit state
$E_{cm}$	Secant modulus of elasticity of concrete
$E_s$	Modulus of elasticity of reinforcing (prestressing steel)
$G_k$	Characteristic permanent load
$I_c$	Second moment of area of concrete
$M_{bal}$	Moment on a column corresponding to the balanced condition design value of moment
$M_{Ed}$	Design value of moment

$M_u$	Ultimate moment of resistance
$Q_k$	Characteristic variable load
$b_w$	Minimum width of section
$f_{ck}$	Characteristic cylinder strength of concrete
$f_{cm}$	Mean cylinder strength of concrete
$f_{ctm}$	Mean tensile strength of concrete
$f_{yk}$	Characteristic yield strength of reinforcement
$g_k$	Characteristic permanent load per unit area
$k_1$	Average compressive stress in the concrete for a rectangular parabolic stress section
$k_2$	A factor that relates the depth to the centroid of the rectangular parabolic stress section and the depth to the neutral axial
$l_a$	Lever arm factor = $z/d$
$q_k$	Characteristic variable load per unit area
$n$	Modular ratio
$\psi$	Action combination factor
$\gamma_c$	Partial safety factor for concrete strength
$\gamma_G$	Partial safety factor for permanent loads, $G$
$\gamma_Q$	Partial safety factor for variable loads, $Q$
$\gamma_s$	Partial safety factor for steel strength
$\delta$	Moment redistribution factor
$\varepsilon$	Strain
$\sigma$	Stress
$\varnothing$	Bar diameter
$b_{eff}$	Effective width of the concrete flange
$E_s$	Modulus of elasticity of steel
$E_{c,eff}$	Effective modulus of elasticity of concrete
$E_{cm}$	Secant modulus of elasticity of concrete
$F_{cm}$	Mean value of the axial tensile strength of concrete
$h$	Overall depth (thickness)
$h_f$	Thickness of the concrete flange
$\delta$	Deflection at mid span
$\gamma$	Factor of safety

## LIST OF APPENDICES

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