Development of Mathematical Model to Decide the Optimal Graphite Product Mix to Enhance the Profit

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Abstract
Sri Lanka is the only country in the world known to extract and produce commercially viable quantities of natural crystalline vein graphite. Currently, Bogala graphite mines and Kahatagaha graphite mine are famous underground mines and the largest natural graphite producers, which supply natural vein graphite in the form of various product categories to the international graphite market. Out of that two major mines, Bogala graphite mines extracts approximately 350 metric tons of natural graphite per month. When it comes to extracted graphite from underground (run of mine), the raw graphite consists of various carbon contents in the form of lumps, chips, and powder. Such graphite is subjected to hand sorting or mechanical separation before it is used for further processing to produce various product categories as requested by local or international customers. During the graphite processing stage, it is a real dilemma to decide that with available graphite in raw form in which carbon content varying from 80% to 99%, to produce saleable product to which customer orders with a view to getting maximum profit out of various pricing for various saleable graphite products that Bogala graphite mine produces. So, it is worth finding out which product mix gives the highest monthly revenue utilising its limited monthly underground mine production and limited machine capacities. The problem addressed here is to determine the product mix (combination of sales package) to be adopted by the company for selling her graphite products at which the optimal profit level would be attained.

Keywords: Carbon, Crystalline vein graphite, Processing, Run of mine, Underground

1. Introduction

Crystalline vein graphite is believed to be a naturally occurring pyrolyte graphite condensed from gas or liquid phase under high temperature and pressure well below in earth crust. Deposits consist of veins of variable thickness and various carbon content ranging from 80% to 99% carbon contents. In Sri Lanka, currently, Bogala Graphite mines and Kahatagaha Graphite mines are famous underground mines and the largest natural graphite producers, which supply natural vein graphite in the form of various product categories to the international graphite market. Out of two major mines, Bogala graphite mines extracts approximately 350 metric tons of natural graphite per month. When it comes to mined out graphite from underground (run of mine), the raw graphite consists of various carbon contents in the form of lumps, chips and powder. Such graphite is subjected to hand sorting or mechanical separation before it is used for further processing to produce various product categories as requested by local or international customers.

Below diagram is useful to identify the production flow and the machineries used for preparing each graphite grade. The inter-relation of the machinery can also be clearly understood with that schematic diagram. Final graphite product grades which are manufactured using different machines, are shown in the diagram as well.

Schematic diagram of Production Flow of Graphite Processing in Bogala Graphite Mines

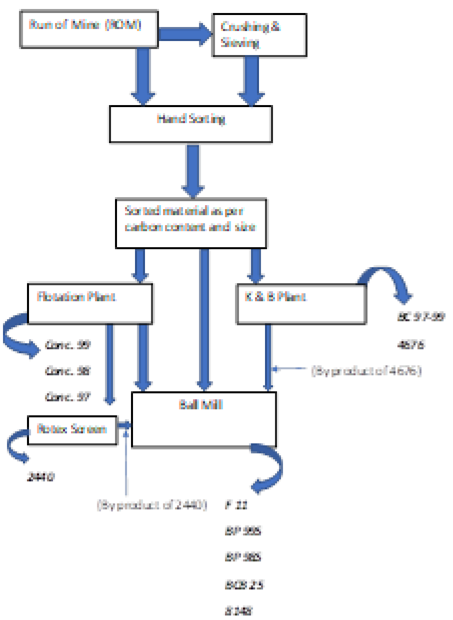

Fig. 1: Major steps in Graphite Processing

During the graphite processing stage, it is a real dilemma to decide that with available graphite in raw form in which carbon content varying from 80% to 99%, to allocate raw material to which customer orders with a view to getting maximum profit out of various pricing for various graphite products that Bogala graphite mine produces. So it is worth finding out which product mix gives the highest monthly revenue utilizing its limited monthly underground mine production, which is 350 mt. The problem addressed here is to determine the product mix (combination of sales package) to be adopted by the company for selling her graphite products at which the optimal profit level would be attained.

1.1 Literature Review

It is generally agreed that the use of mathematical algorithms came into existence as a discipline during World War II when there was a critical need to manage scarce resources. However, a particular algorithm and technique can be traced back as in world war-I, when Thomas Edison (1914-1915) made an effort to use a tactical game board for a solution to minimize shipping losses from enemy submarines instead of risking ships in actual conditions [2]. So under this heading, we shall review the existing recent literature which are related to the topic furthermore.

According to Mille in 2007, linear programming is a generalization of linear algebra used in modelling so many real-life problems ranging from scheduling airline routes to shipping oil from refineries to cities for the purpose of finding inexpensive diets capable of meeting daily requirements. Miller argued that the reason for the great versatility of linear programming is due to the ease at which constraints can be incorporated into the linear programming model [4]. Linear programming techniques have been used in many industrial applications with a view to getting optimal solutions in different requirements [1]. The beverage industry, Oil refinery, Agriculture, manufacturing of different product categories using same raw material, energy sector, Facility location, vehicle routing and scheduling, personnel, machine and job scheduling, product mixes, and inventory management are a few fields where LP models applied to get optimal solutions for achieving highest profit, maximum use of scarce raw material and efficient output.

So the summary of Linear Programming model formulation steps for a product mix company can be given as;

1. Define the decision variables,
2. Define the objective function to maximize profit, and
3. Define the constraints.

With that understanding, it is possible to go for good decision-making to maximize the profit of a company engaged in manufacturing different product mixes.

When it comes to the application of operation research or mathematical modelling in mine planning and product mix optimization for maximizing profits using mathematical modeling, it dates back to the 1960s [3]. Since that time, optimization and simulation, in particular, have been applied to both surface and underground mine planning problems, including mine design, long and short-term production planning, equipment selection and dispatching etc.

2. Methodology

Bogala Graphite Lanka Plc. is chosen for this research study for two main reasons. First, it uses natural graphite mined as run-of-mine from its own underground mine and is used for producing various graphite products by varying its carbon content and particle size and some other physical properties. Secondly, Bogala Graphite mainly exports many different graphite products to many international customers with different price categories, which decide the profits to the company based on profit margins of each product category.

Primary/ Secondary data that is going to be used are as follows, and all that was taken from the available records in Bogala Graphite Lanka PLC.

- Carbon grade-wise quantities sorted from monthly production.
- Product categories processed as per orders from buyers and prices.
- Possible product grades to be processed using available raw material or sorted material.
- By-product information when preparing each product category.

Considering the constraints of raw material available for a month and underground production, which is always limited, a mathematical technique such as linear programming would be much useful to solve and obtain optimum solution for product mix. Linear Programming is a mathematical technique for generating and selecting the optimal or the best solution for a given objective function. It may be defined as a method of optimizing (i.e., maximizing or minimizing) a linear function for a number of constraints stated in the form of linear equations. Generally, the objective function may be a maximization of profit (which is the focus of this research work) or minimization of costs or labor hours. Moreover, the model also consists of certain structural constraints, which are a set of conditions that the optimal solution should justify. Examples of structural constraints include raw material constraints, production time constraints, and skilled labour constraints, to mention a few.

The general linear programming problem (or model) with *n* decision variables and *m* constraints can be stated in the following form.

(Max or min) $z = c_1x_1 + c_2x_2 + \ldots\ldots + c_nx_n$
(Objective function) -----------------> (1)

Subject to linear constraints of the form

$a_{11}x_1 + a_{12}x_2 + \ldots\ldots a_nx_n \ (\leq, =, \geq)\ b_1$

$a_{12}x_1 + a_{22}x_2 + \ldots\ldots a_{2n}x_n \ (\leq, =, \geq)\ b_2$

……………………………………………………

……………………………………………………$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots a_mx_n \ (\leq, =, \geq)$

$x_1 + x_2 + \ldots\ldots + x_n \quad \geq 0$

Model Assumptions made for the analysis are as follows.

a. The production quantity is the same as the sales. As it is unrealistic to produce what is not sold.
b. There is a linear relationship among the variables used in the model.

2.1 Data Collection and Analysis

The data for this research study are in the below tables which were extracted from the production and sales records of Bogala Graphite Lanka Plc. In Bogala graphite mine, the average monthly underground mine production (run of mine or ROM) is 400 metric tons, and for this research, that figure has been assumed to be fixed for all calculations. Once sorted by manual curing, the composition of ROM according to carbon % is as follows.

Table 1: Composition of run of mine.

Carbon %	Composition	Form
99+	1%	Lumps
97-99	21%	Lumps
90-97	16%	Lumps/ chips
70-90	48%	Tub dust
Below 70	11%	Tub dust
Pure rock	3%	

Product grade requested by buyers and its Carbon %, profit margins (assumed values) and demand in metric tons (assumed values and varies monthly) and available machine hours of each plant per month are given in below two tables.

Table 2: Product grade with C%, profit margin and demand.

Product Grade	Carbon Content	Profit (Euro)/ mt	Demand Qty. (mt)
2440	99%	1329	40
BP99S	99%	1241	25
Conc. 99	99%	717	60
BC 97 99	97-99%	597	40
4676	97-99%	559	60
8148	97-99%	517	20
BP98S	97-99%	510	25
BCB 25	97-99%	414	30
F 11	97-99%	339	40
Conc. 98	97-99%	267	60

Table 3: Machine hours availability for each plant

No	Product grade	Plant used	Machine hrs reqd. / mt
1	F 11	Ball mill	2 ½
2	2440	Rotex Screen	1 ½
3	BC 9799	K & B Plant	¾
4	BP 99 S	Ball mill	3
5	BP 98 S	Ball mill	2 ½
6	BCB 25	Ball mill	2 ½
7	8148	Ball mill	2 ½
8	4676	K & B Plant	1 ½
9	Conc. 99	Flotation	1 ¾
10	Conc. 98	Flotation	1 ½
11	Conc. 97	Flotation	1 ½

Formulation of Objective function:

Maximize profit = (1329) X1 + (1241) X2 + (717) X3 + (597) X4 + (559) X5 + (517) X6 + (510) X7 + (414) X8 + (339) X9 + (267) X10

Where X1, X2,…. X10 will give the possible production quantity of each graphite grade in making an optimum profit with respect to the graphite grades 1 to 10 in the above table.

Decision variables and Constraints were identified for available raw material and available machine hours in a particular month. That information was used for MS Excel Solver with a view to analyse the best solution for the situation.

The aim of the analysis is to use the above data in a suitable linear programming model so that to find the product mix that gives the highest profit margin.

3. Results

By using MS Excel Solver, the product mix that gives the highest profit margin against the sales demand can be found. So the company can go for the highest profit with that mix, and the changes of raw material grades in the run of mine can change the mix in each month. In such a situation, this model can be run with input changes so that each month different product mix values can be taken to achieve the highest profit margin.

Answer report given by Excel Solver for this particular analysis shows the required mix of product grades which gives the highest profit margin against the product demand of that particular month.

Microsoft Excel 16.0 Answer Report
Worksheet: [LP model.xlsx]Sheet1
Report Created: 7/06/2021 10:16:00 AM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0.031 Seconds.
Iterations: 9 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Max)

Cell	Name	Original Value	Final Value
I17	Z = Lumps/chips	0	82994.20922

Variable Cells

Cell	Name	Original Value	Final Value	Integer
E18	X1 Demand	0	60	Contin
E19	X2 Demand	0	50	Contin
E20	X3 Demand	0	60	Contin
E21	X4 Demand	0	40	Contin
E22	X5 Demand	0	30	Contin
E23	X6 Demand	0	50	Contin
E24	X7 Demand	0	30	Contin
E25	X8 Demand	0	40	Contin
E26	X9 Demand	0	20	Contin
E27	X10 Demand	0	20	Contin

Figure 2: Answer report given by MS Excel Solver

4. Discussion

An algorithm of Linear Programing with the support of MS Excel Solver was used to solve the product mix problem. This algorithm solved the linear program problem and gave the optimal product mix against the sales demand of customers so that the company can decide how to promote and sell to produce the optimal product mix that gives the highest profit in a particular month.

Optimization problems in many fields can be modeled and solved using Excel Solver as done in this particular scenario. It does not require knowledge of complex mathematical concepts behind the solution algorithms.

5. Conclusion

The objective of this study was to apply a suitable mathematical algorithm or technique to optimize profit margin against sales demands for various graphite product grades in a particular month. For solving this problem, a linear programming model using MS Excel Solver has been developed. The Linear programming technique determined optimum profit values for each graphite grade demanded by customers and gave the product mix, which gives the highest profit.

Based on the findings of the research, the following conclusions can be made:

1. Optimal product mix giving highest profit in selling graphite products can be achieved with the application of this linear programming model. The system of equations can be expanded or reduced to accommodate any variety of system combinations.

2. The research is significant in the sense that it will assist the company in making corrective decisions well in time using the methods of linear programming. This will determine the future production patterns and outlook resulting in the establishment of new production grades while planning for sales profit maximization of the company.

3. Although this research deals with maximizing profit, it can be applied to other process industries with similar production processes.

So manufacturing companies can develop linear programming models to help decide how many units or quantities of different products they should produce to maximize their profit (or minimize their cost), given scarce resources such as raw material, machine hours and available labour.

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