AN INVISCID MODEL FOR PREDICTING UNSTEADY FORCES IN DOUBLY CONNECTED DOMAINS

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March 2016

DECLARATION OF THE CANDIDATE & SUPERVISOR

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Abstract

Inviscid analytical-numerical model for predicting unsteady forces on two aerofoil configurations is developed and validated with the past literature. First the unsteady inviscid, incompressible and irrotational, except the logarithmic singularities at vortex points, flow field around the doubly connected domain is evaluated using a conformal mapping method. A discrete vortex shedding mechanism is incorporated to model the free shear layers of the real fluid flow. The complex potential associated with uniform flow and the vortex motion is obtained using elliptic functions and the modified Green's function respectively. The strengths of the vortices are evaluated using Kutta condition which keeps the regularity of the flow field. Circulation development around the aerofoils is quantified by utilizing Kelvin's circulation theorem. The unsteady forces are obtained using the unsteady version of the Blasius equation. Both trapezoidal rule and finite difference method are incorporated to solve the unsteady Blasius equation. The developed inviscid model is applied to various aerofoil configurations to predict the unsteady forces on the aerofoils. The results obtained were validated to the past relevant literature. Results showed a good agreement with the past literature.

Keywords: unsteady, forces, plunging, analytical-numerical, doubly

The work is dedicated to my parents

ACKNOWLEDGEMENT

First of all I would like to thank my parents for the support they have given me throughout the research. I would also like to thank my supervisors, Dr. W.K. Wimalsiri and Dr. V.P.C. Dassanayake, the research coordinator, Dr. R.A.R.C. Gopura, chairperson, Dr. R.A.C.P. Ranasinghe, examiners, Dr. A.G.T. Sugathapala and Dr. M. Narayana for their support and the guidance. I would also like to thank Dr. Nirosh Jayaweera for providing me with a high performing computer.

My special thanks go to Mr. W.P.D. Welgama for his enormous support given to me for the completion of the research. I also would like to thank Mr. K.K.M.N.P. Samaraweera, Prof. Ismail H. Tuncer and Prof. Frank Stenger for the fruitful talks about the study. I would like to thank Mr. Malinga Silva, Mr. Vishva Jith Wijayasundara, Mr. Nadun Vithanawasam, Mr. Ramesh Gunaratna and Mr. Deshan Goonethilleke for financially supporting me throughout the last nine months of the research and Mr. Sasiranga de Silva, Dr. Ranjith Amarasinghe, Mr. Charith Perera, Mr. Brayan Livera, Mr. Madura Peiris, Mr. Gayan Sirimanna and Mr. Indika Vithana for their support.

I am in debt to Mr. M.A.R.T. Gunarathna and R.P. Gunarathna for the learning material they provided in early stages of the research and I would also like to thank Dr. Thilina Lalitharathna for showing me a way to reach for learning material in an easy manner which was really time saving.

Finally, I would like to thank the non-academic staff members in Thermodynamics lab and the controls lab in the department of mechanical engineering for facilitating me with the things I needed.

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