

**MODELING AGEING POPULATION IN SRI LANKA  
TOWARDS BETTER CARE FOR ELDERLY**

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Degree of Master of Science

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Master of Science in Business Statistics

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July 2017

## **DECLARATION**

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## **ABSTRACT**

Population ageing is an universal phenomenon and it is expected to be among the most prominent global demographic trends of the 21<sup>st</sup> century. In Sri Lanka there was a rising trend of ageing population throughout the past years and has recorded the highest number of agers within South Asia. However, no sound statistical or mathematical models were developed to project ageing population in Sri Lanka. Using the population aged 60 years and above in Sri Lanka during 1950-2016, three types of statistical models: (i) ARIMA (0, 2, 1), (ii) growth model, and (iii) double exponential smoothing model were developed. The models were compared using various statistical indicators and some statistical diagnostics tests. The comparison was done for both training set as well as validation set. Among these models the double exponential smoothing model was found as the best fitted model. According to the forecast derived from the best fitted model, it was found that the increasing trend of ageing population in the country will continue in the future and there will be approximately 2,936,000 ageing population in Sri Lanka in 2020. The information obtained in this study is beneficial for planners and decision makers in the government sector and other relevant organizations to cater to the needs of the increasing agers in the future of Sri Lanka.

**Keywords:** Ageing Population, ARIMA, Demographic, Exponential Smoothing

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## LIST OF ABBREVIATIONS

Abbreviation	Description
ACF	Autocorrelation Function
AIC	Akaike's Information Criteria
ARIMA(p,d,q)	Autoregressive Integrated Moving Averages of order p, d, and q,
ARMA(p,q)	Autoregressive of order p and Moving Averages of order q
AR	Autoregressive components
MA	Moving averages components
BIC	Schwarz Information Criterion
CBR	Crude Birth Rate
CDR	Crude Death Rate
DDT	<i>dichloro diphenyl trichloroethane</i>
DES	Double Exponential Smoothing
$\rho_k$	Autocorrelation function of lag k
$\varphi_{kk}$	Partial autocorrelation function for lag k
MAPE	Mean Absolute Percentage Error
SACF	Sample Autocorrelation Function
TFR	Total Fertility Rate

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