DEVELOPMENT OF A DIRECT EXCHANGE AREAESTIMATION ALGORITHM FOR RECTANGULAR ENCLOSURES

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DECLARATION

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ABSTRACT

Conduction, convection and radiation are the principle modes of transferring heat from a source to sink. In a furnace, when the operating temperature is above 1000^0C, radiation heat transfer will be the predominant mode of transferring the heat. For such a furnace, estimating the radiation heat transfer accurately is essential. To estimate the heat transfer in the absence of participating medium, obtaining radiation properties of surfaces, temperatures of surfaces and view factor between surfaces is necessary. When participating media is concerned, it mainly consists of CO_2 and H_2O due to fossil fuel combustion. Mixtures of these gasses are typically participating in radiation heat transfer process with different emissivity values at different temperatures. Therefore, view factor concept will not be accurate in such an instant. Direct Exchange Area (DEA) is introduced to cater for participating medium involved problems. To evaluate the radiation heat transfer with participating medium using zonal method, it is essential to determine the DEA values. Direct integration and Monte Carlo are the main methods to evaluate view factor. However, Monte Carlo method is not viable to apply for simple shapes such as rectangular enclosures due to large time consumption and computer storage requirement. Thus, direct integration is a good approach to find DEAs for simple geometries.

Surface to surface zones (**SS)**, surface to volume zones (**SG)**, volume to volume zones (**GG**) are the scenarios which needed to be determined in DEA estimation. Generalized mathematical equations for DEAs were simplified by using vector algebra with considering the simple shape of rectangular furnace walls. Further simplification could be done by reducing the integration scheme. Later, computer programming has been used to estimate the DEA values which is based on numerical techniques.

The resulting programming code is based on Matlab software, which has been developed to determine DEAs for each and every zones surface and volume which will be decided by the user. Estimated DEAs are not available in literature due to its dependency on area and absorption coefficient variable selected by the user. However, programming code based results validation can be done in two ways. One way is to convert DEA values for surface zones in to view factors by avoiding the effect from participating medium. Then the evaluated results can be compared with well known literature. Secondly, a mathematical relation which will be explained in literature can be used to compare overall results.

The computer based program was sophisticated with a user friendly interfaceso that the user or the designer does not need to worry about what is happening inside. The calculated result will later be transformed in to matrix form which can be directly used on estimation of heat transfer.

For future work, improvement of developed software interface to perform in optimum condition, enhancing with more features to cater for scattering situations, handling of garbage input by user, reducing the effect on higher absorption coefficient on the results were suggested.

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NOMENCLATURE

A	=	Area
b1,b2,b3	=	constants
E	=	Energy flux
Fij	=	View factor from i surface to j surface
g	=	gas
H	=	irradiation
J	=	radiosity
k	=	transmissionfactor
q	=	Heat flux
s	=	Vectorized distance
S	=	Scalar distance
V	=	Volume
w	=	Wall
x	=	direction of x
y	=	direction of y
z	=	direction of z

Greek Letters

α	=	thermal diffusivity in convection / absorptivity in radiation
ρ	=	Transmissivity
μ	=	kinematic viscosity
ε	=	Emissivity
ε	=	Emissivity matrix
σ	=	Stefan Boltzmann constant
γ	=	Reflectivity
τ	=	Transmissivity

LIST OF ABBREVIATIONS

DEA	Direct Exchange Area
CPU	Central Processing Unit
VF	View Factor
SS	Surface to Surface DEA factor matrix
GS	Gas volume to surface DEA factor matrix
GG	Gas volume to Gas volume DEA factor matrix

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1. INTRODUCTION

1.1 Background

When there is temperature difference between two surfaces, thermal energy always tries to transferfrom one surface to other surface to make the system balance. This phenomenon is known as heat transfer and net heat transfer is the term for the result of attempting to balance the thermal system. Conduction, convection and radiation are the three mechanisms of transferring thermal energy from one surface to other. Each mechanism has unique way of transferring heat.

Heat treatment of an object / work piece to achieve desired properties, is one major application of heat transferring in industry[1]. Furnace or kiln is the term which used to denote the enclosure which is used to achieve above mentioned requirements. “In most high temperature furnaces operating at above 1000^0C, about 90% of the heat transfer is being carried out by radiation heat transfer mechanism”. Ceramic, bricks and pottery making kilns (operates around 1000^0C), cement and lime kilns (operates around 850^0C – 1000^0C), Glass making furnaces (operates around 1400^0C – 1600^0C) are some examples of such applications[2]. Therefore, it is very important to determine radiation heat transfer accurately to get overall picture of the furnace performance. In order to determine radiation heat transfer between two surfaces, it is necessary to uncover the information about the temperatures of the two surfaces, radiation properties of both surfaces (eg: emissivity, absorptivity,.), the direct exchange area factors (generalized version of view factors) and the radiation properties of participating medium[3]. When estimating the radiation heat transfer between two surfaces, estimation of view factor is the most difficult task which is needed to be carried out at side. If pre calculated view factors are not available, even with the simplest shape where two surfaces are involved, it will be difficult to find a solution due to mathematical complexity of the view factor calculation. Strictly speaking, view factor is bounded by the condition of none participating medium or the behavior of the medium is ideal. In practically, combust air (for direct fired furnaces) contains of CO2 and H2O, whose emission and absorption coefficients are different at different temperatures[4]. Therefore, usage of view factor is no longer valid for such instances. Direct exchange area (DEA) is the term which is introduced to overcome such issues related with view factor (area dependent, participating medium)[5]

DEA simply means, how a volume or surface sees the other surface or volume. DEA estimation is even more complicated than view factor estimation due to the appearance of participating medium. When it comes to a furnace or kiln, it will become multiple surfaces which interact with others and its own by means of radiation heat exchange. Therefore, determination of DEA for each and every surface with participating medium is necessary. Furthermore, furnaces are having large wall areas and more than one heat source, which will lead for variation in the wall temperature along the surface. Therefore, the surface has to be divided in to several isothermal segments in evaluation. This procedure can be further extended to incorporate volume zones as well. Analyzing an enclosure with several zones is known as zonal method. Analyzing several zones will direct the DEA estimation into complex, repetitive computational estimation requirement with furnace walls as in its boundaries.

1.2 Problem statement

For a furnace designer or a person who search for improvement of a furnace, should find out the amount of heat energy transferred through radiation. In order to do so, it is essential to determine DEA values on the side. Estimation of DEA is a difficult task due to involvement of complicated mathematics. However, this needed to be done several times to identify suitable conditions of the furnace to move for an optimized solution although is a tedious task. There is a requirement for cater this matter to aid furnace design / optimization process according to their variable inputs.

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1.3 Aim

To eliminate the tedious task of estimating DEA repeatedly and accurately according to the designer's or furnace investigator's dynamic requirement.

1.4 Objectives

- To developing a computable mathematical model using integral reduction and numerical techniques to estimate DEA.

- To integratemathematical model to obtain DEA values for rectangular enclosure.

- To develop auser friendly system, which can be used to estimate all DEAs while maintaining the flexibility to adopt according to the user defined inputs.

2. LITERATURE REVIEW

2.1 Radiation heat transfer

Any object emits thermal energy through radiation no matter what temperature it holds. This is done by means of electromagnetic radiation mechanism. However, the concerning part is the net radiation heat transfer which will cause for change of the temperature and the properties of the object. Although the object emits different electromagnetic waves in the complete spectrum, wave lengths between 10^{-7}m and 10^{-3}m plays the vital role in radiation heat transfer[6]. Amount of heat flux which emits by a particular body at particular temperature has been prescribed by the Stefan – Boltzmann low, given by

$$E = \sigma T^4 \qquad \text{Eq . 2.1}$$

This is an ideal case where the emissive body is called as a black body. Another property of black body is the capability of absorbing all incoming radiation. In this case reflectance and transmittance of energy of the incident radiation becomes zero. Some typical cases of different types of bodies are summarized below.

$$\alpha = 1, \rho = 0, \tau = 0 \quad Blackbody$$

$$\rho = 1, \alpha = 0, \tau = 0 \quad Whitebody$$

$$\rho < 1, \alpha < 1, \tau \geq 0 \quad Greybody$$

$$\rho < 1, \alpha < 1, \tau = 0 \quad Greyopaquebody$$

$$\alpha = \varepsilon Forthermallyequilibriumbodies$$

Although the last equation for the relation between absorptivity and emissivity is proven by Kirchhoff's low, the validation will still holds if the surfaces are diffuse [6]. When a furnace is considered, it has walls which are supposed to be well insulated and the participating gas medium which consists of combustion gas. Majority of the furnace surface insulation material are considered as diffuse. Furthermore, for gaseous medium, it can be assumed as a grey gas relation for most cases[2]. However, Hottel and Egbert (1942) came up with an empirical relation to determine absorption coefficient using emissivity of the gas [7].

$$\alpha_g(t_g, t_w) = \left[\left\{1 - \left(1 - \varepsilon_g(t_w)\right)\right\}^{\frac{t_g}{t_w}}\right]\left[\frac{t_g}{t_w}\right]^n \qquad \text{Eq. 2.2}$$

Where n = 0.65 for CO2 and n = 0.45 for H2O. With these factors overall absorptivity can be estimated. This estimation is more accurate if the furnace is gas fired. If the furnace is liquid fuel based or coal based, properties of fuel particles and properties of soot particles should also be determined during the overall absorption coefficient estimation. Nevertheless, these data are readily available for various temperatures and pressures.

Incorporating the emissivity in to the equation 2.1 leads to

$$E = \varepsilon\sigma T^4 \qquad \text{Eq. 2.3}$$

This energy flux will emit over hemisphere around its position as the centre. More useful application of radiation heat transfer is to determine the net heat exchange between two surfaces rather than the emission or absorption of one particular surface.

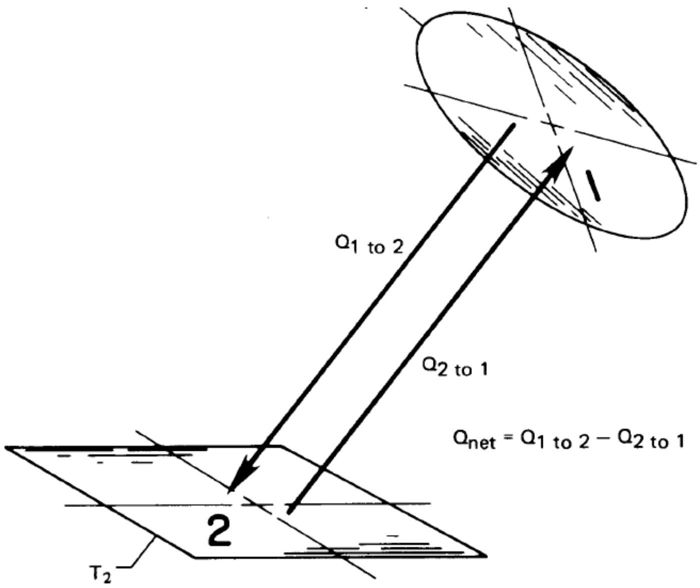

Figure 2.1: Radiation heat exchange between two surfaces

(source: [3])

Consider the arbitrary selected geometrical surfaces as shown in figure 2.1. Surface 1 emits heat flux in the whole hemisphere surrounding itself. The total energy emitted by the surface depends on the area of the emitting surface. It can be observed that only a part of the emitted energy from the surface 1 can be received by surface 2. Furthermore, when the area of two surfaces are changed, the amount of heat which can be received will also be altered. The fraction of energy which can be received by surface 2 from surface 1 is known as view factor. The F_{1-2} is the notation which implies fraction of radiant energy emitted by surface 1 and intercepted by surface 2. The net heat transfer between two grey surfaces (no participating medium) is expressed by considering incident radiation from other surface, emitted radiation from the surface due to its body temperature and reflected incident radiation.

$$E_{net,1} = A_1(J_1 - H_1) \qquad \text{Eq. 2.4}$$

Where

$$J_1 = E_1 + \rho_1 H_1 \quad [10] \qquad \text{Eq. 2.5}$$

By generalize the equation to multiple surfaces, it will end up with

$$q_i = \frac{\varepsilon_i}{1-\varepsilon_i}(E_{b,i} - J_i) \qquad \text{Eq. 2.6}$$

$$\frac{q_i}{\varepsilon_i} - \sum_{j=1}^{N}\left(\frac{1}{\varepsilon_j} - 1\right) F_{i-j}\, q_j + H_{o,i} = \sum_{j=1}^{N} F_{i-j}\,(E_{b,i} - E_{b,j})[5] \qquad \text{Eq. 2.7}$$

It can be seen from Eq. 2.7 estimation of view factor between each and every surfacesis a necessity to estimate heat flux of every surface.

2.2 View factor

The mathematical definition for view factor is as follows

$$F_{i-j} = \frac{1}{A_i}\int_{A_i}\int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi S^2}\, dA_j\, dA_i \qquad \text{Eq. 2.8}$$

Thus, estimation of view factor needs more mathematical treatment to reduce its complexity. Furthermore, this process should repeat for each and every case as mentioned in earlier. However, from reciprocity relation of view factors

$$A_i F_{i-j} = A_j F_{j-i} \qquad \text{Eq. 2.9}$$

With this relation, number of estimation reduced from N x N to N (N-1)/2. Still the task remains up to a certain level although the computational time is reduced.
View factors estimation can be done by different ways and means. A popular classification of estimation methods is given below.

Direct integration method

- Surface integration
- Contour integration

Monte carlo method

Special methods

- View factor algebra
- Crossed strings method
- Unit sphere method
- Inside sphere method[5]

Comparison of above mentioned approaches is given below Table 2.1

Table 2.1: Comparison of view factor calculation methods

Direct integration method	Monte carlo method	Special methods
Analytical or numerical approach is involved	Statistical approach is involved	Complex mathematical treatment is avoided due simple geometry or availability of one or two view factors
Possible when the view factor equation for the surface can be represented by Cartesian, polar, spherical coordinates	Possible for complex geometries as well as simple geometries.	Not possible when the shape deviates from simple geometries such as rectangles, circles and triangles
Participating medium consideration is possible	Participating medium is possible	Participating medium is impossible
Consideration of radiation scattering is not possible or rather complex	Consideration of radiation scattering is possible	N/A
Anisotropic material consideration is not possible	Anisotropic material consideration is possible	N/A

Table 2.1 (cont.)

Direct integration method	**Monte carlo method**	**Special methods**
Becomes complex when obstructs are involved in between surfaces	Investigation with obstructed objects is possible	N/A
Computing is cheap with respect to Monte carlo	Computationally expensive due to storage requirements	Computational techniques may not require due simple calculations.
Time consumption is small for simpler shapes. But significant for complex.	Time consumption does not significantly or exponentially vary with the shape	N/A
Numerical approach might encounter an error specially with close segments.	Sampling error is occurring as a noise	Minimum error.

By repeating Eq. 2.8

$$F_{i-j} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{cos\theta_i cos\theta_j}{\pi S^2} dA_j \, dA_i$$

With usual vector notation, it can be determined the vector for the segment i to j

$$s_{ij} = (x_j - x_i)\hat{\imath} + (y_j - y_i)\hat{\jmath} + (z_j - z_i)\hat{k} \qquad \text{Eq. 2.9}$$

And the normal vector

$$\hat{n} = l\hat{\imath} + m\hat{\jmath} + n\hat{k} \qquad \text{Eq. 2.10}$$

Length of the vector between i and j is given by

$$\left|s_{ij}\right|^2 = \left|s_{ji}\right|^2 = S^2 \qquad \text{Eq. 2.11}$$

By considering vector algebra following equations can be achieved

$$cos\theta_i = \frac{\hat{n}_i.s_{ij}}{S} = \frac{1}{S}\lfloor((x_j - x_i)l_i + (y_j - y_i)m_i + (z_j - z_i)n_i\rfloor \text{Eq. 2.12a}$$

$$cos\theta_j = \frac{\hat{n}_j.s_{ji}}{S} = \frac{1}{S}\lfloor((x_i - x_j)l_j + (y_i - y_j)m_j + (z_i - z_j)n_j\rfloor \text{Eq. 2.12b}$$

With applying these relations to eq. 2.8, it can be converted in to quadruple line integrations with applicable limits.

2.3 DEA factor

According to the definition of view factor equation 2.8, it can be seen that the integrated result is divided by area A_i . When a participating medium is present what will happen to this equation?. Furthermore, there will heat exchange between surface to volume and volume to volume. In that case defining a view factor for a gas is not possible. Because, the gas will have the shape of its containment. To avoid such difficulties, new concept called DEA was introduced[8]. Where it is defined by

$$\overline{s_i s_j} = \overline{s_j s_i} = \int_{A_i}\int_{A_j}\frac{cos\theta_i cos\theta_j}{\pi S^2}dA_j dA_i \qquad \text{Eq. 2.13}$$

This technique can be further extended to cater for the participating medium resulting following equations.

$$\overline{s_i s_j} = \overline{s_j s_i} = \int_{A_i} \int_{A_j} e^{-ks} \frac{cos\theta_i cos\theta_j}{\pi S^2} dA_j \, dA_i \text{ for surface to surface} \quad \text{Eq. 2.14}$$

$$\overline{g_i s_j} = \overline{s_j g_i} = \int_{V_i} \int_{A_j} e^{-ks} \frac{cos\theta_j}{\pi S^2} k dA_j \, dV_i \quad \text{for surface to volume} \quad \text{Eq. 2.15}$$

$$\overline{g_i g_j} = \overline{g_j g_i} = \int_{V_i} \int_{v_j} e^{-ks} \frac{k^2}{\pi S^2} dv_j \, dV_i \quad \text{for volume to volume} \quad \text{Eq. 2.16}$$

It is obvious that, when there is no participating medium, k becomes zero. This will leads to all $g_i s_j$ values and $g_i g_j$ values to be zero and Eq. 2.14 becomes Eq. 2.13. Furthermore, transmission factor e^{-ks} becomes 1 indicates that all the emitted radiation from one surface towards other surface will receive without any interference. The appearance of participating medium will make the analytical approach more difficult. It definitely needs mathematical treatment to reduce its complexity.

There are many literatures to obtain view factors between perpendicular surfaces and parallel surfaces in graphical format. Appendix 1 shows such example for view factor estimation.

2.4 Zonal Method

With the knowledge of Eq 2.14, 2.15, 2.16, DEA factors can be estimated. If a furnace is considered, there are different sections inside that large volume. Heat generating or heat incoming section, combust gas emission to the chimney section, combust gas leaking to the environment sections, object locating section are some examples of different sections with different temperatures. Therefore, estimation of heat transfer between two walls by assuming the wall as one complete surface or estimation of heat transfer between the complete volumes to wall is not viable. Furthermore, according to the Eq 2.2 it can be seen that absorptivity can be changed according to the temperature. Not only that, but also emissivity tends to change with the temperature and pressure [9]. Due to these reasons, there is a necessity to divide the furnace into isothermal areas or volumes where the thermal properties remain unchanged. This method is called zonal method and it was developed[10]. According this method complete enclosure is divided into number of isothermal surfaces and volumes. The heat transfer between each every surface and volumes will be accounted separately. This will leads to several sets of equations for surface to surface, surface to volume, and volume to volume cases.

Incorporating eq. 2.14, eq. 2.15 and 2.16 into zonal method will leads to following set of equations for ith volume zone and surface zone

$$Q_{si} = \varepsilon_i A_i E_{bsi} - \sum_{j=1}^{N} \overline{S_i S_j}\, E_{bsj} - \sum_{m=1}^{K} \overline{S_i G_k}\, E_{bgm} \quad \text{Eq. 2.17}$$

$$Q_{gi} = 4kV_i E_{bgi} - \sum_{j=1}^{N} \overline{G_i S_j}\, E_{bsj} - \sum_{m=1}^{K} \overline{G_i G_k}\, E_{bgm} \quad \text{Eq. 2.18}$$

Considering the fact that DEA does not depend on the temperature and for a isothermal enclosure, Eq 2.17 and 2.18 will convert to

$$A_i = \sum_{j=1}^{N} \overline{s_i s_j} + \sum_{m=1}^{K} \overline{s_i g_k} \quad \text{Eq. 2.19}$$

$$4kV_i = \sum_{m=1}^{k} \overline{\mathrm{g}_i g_m} + \sum_{j=1}^{N} \overline{s_j g_\mathrm{i}} \quad \text{Eq. 2.20}$$

Where N represents number of surfaces and the m represents number of volumes. In order to proceed more with Eq. 2.17 and 2.18 DEA factors for surface to surface, surface to volume and volume to volume should be found.

$$\boldsymbol{SS} = \begin{matrix} \overline{s_1 s_1} & \cdots & \overline{s_1 s_N} \\ \vdots & \ddots & \vdots \\ \overline{s_N s_1} & \cdots & \overline{s_N s_N} \end{matrix} \quad \text{Eq. 2.21}$$

$$\boldsymbol{SG} = \begin{matrix} \overline{s_1 g_1} & \cdots & \overline{s_1 g_k} \\ \vdots & \ddots & \vdots \\ \overline{s_N s_1} & \cdots & \overline{s_N s_k} \end{matrix} \quad \text{Eq. 2.22}$$

$$\boldsymbol{GG} = \begin{matrix} \overline{g_1 g_1} & \cdots & \overline{g_1 g_k} \\ \vdots & \ddots & \vdots \\ \overline{g_k g_1} & \cdots & \overline{g_k g_k} \end{matrix} \qquad \text{Eq. 2.23}$$

By evaluating SS, SG, GG matrices the task can be fulfilled.

$$\boldsymbol{S} = \boldsymbol{SS}.\boldsymbol{\varepsilon} \qquad \text{Eq. 2.24}$$

$$\boldsymbol{\varepsilon_{bs}} = \begin{matrix} \varepsilon_1 \\ \vdots \\ \varepsilon_N \end{matrix} \qquad \text{Eq. 2.25}$$

Eq. 2.25 will include the emissivity values in to the system.

If a rectangular furnace is considered, definitely it should have six surfaces. For a furnace designer or a person who wish to evaluate the heat transfer performance or for most effective heat transfer zone identification for place the object, he or she should find DEA values of all zones. Furthermore, at design stage, length, width and height of the furnace is an open question, provided that object fits on to the furnace. If one of these changed everything changes.Therefore, there is a necessity for a procedure to obtain SS, SG, GG matrices, while maintaining the repetitiveness, user defined zone size, accurate and user friendly.

3.METHODOLOGY

In order to achieve the objectives following methodology will be proceeded. Based on the literature survey, a suitable DEA estimation technique will be identified for rectangular enclosures. The identified DEA estimation technique will be applied to develop a mathematical model to represent a rectangular enclosure. The developed mathematical model will be converted in to user friendly system while having user requirements as inputs. Finally outcomes will be validated through testing of various input variables.

3.1 Selection of DEA estimation technique

It can be observed that Monte Carlo and direct integration are the available techniques to estimate DEA values which is shown in table 2.1. In order to select one method, features of the selected enclosure should be concerned.

A rectangular enclosure is having following features

- Simple in geometry, only six rectangular surfaces.
- Participating medium should be considered when estimating heat transfer.
- Most of the insulation materials are having isotropic properties.
- Surfaces can be approximated as grey and diffuse.
- Scattering of participating medium can be neglected while having sufficient accurateness
- Participating medium does not have reflectance or considered as grey gas.
- Possible to assume absorption and emission is equal with grey condition[2].

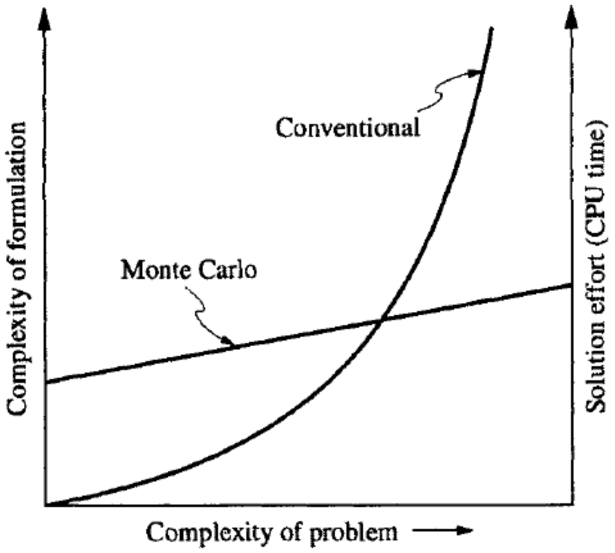

Figure 3.1: Comparison of time consumption between Monte Carlo and conventional methods

(Source : [5])

Having considered table 2.1 and figure 3.1 with above mentioned inherent features of a rectangular type enclosure, it can be justified that conventional method or direct integration method is more applicable to deal with DEA estimation.

3.2 Identification of inputs

An algorithm or procedure to determine SS, SG, GG matrices should be able to satisfy simplicity, repetitiveness, accuracy dependent and user friendliness requirements. Therefore, identification of inputs for such system is an essential feature. Consider the equations 2.14, 2.15 and 2.16. It is evident that whole set of equations are dependent of k value and zone coordinates (s, cosθ, A, V). Therefore,
k value should be considered as an input to the system. By varying k value will introduce different status of the enclosure. Secondly, introducing zone coordinate as an input to the system will make sure variation of zones. Zone coordinates can be

obtained by considering the total enclosure dimensions (in x, y, z axis) and number of divisions along each dimensions (in x, y, z). By having both enclosure dimensions and number of divisions as inputs to the system will make sure that the derived algorithm will have the capability cater for user requirements in flexible manner.

3.3 Derivation of computable equations

Firstly, basic equations 2.14, 2.15, 2.16 should be converted in to simpler integral form by using vector algebra. The set of equations should be able to represent the complete furnace with its boundary conditions.

Consider figure 3.1 which has several zones.

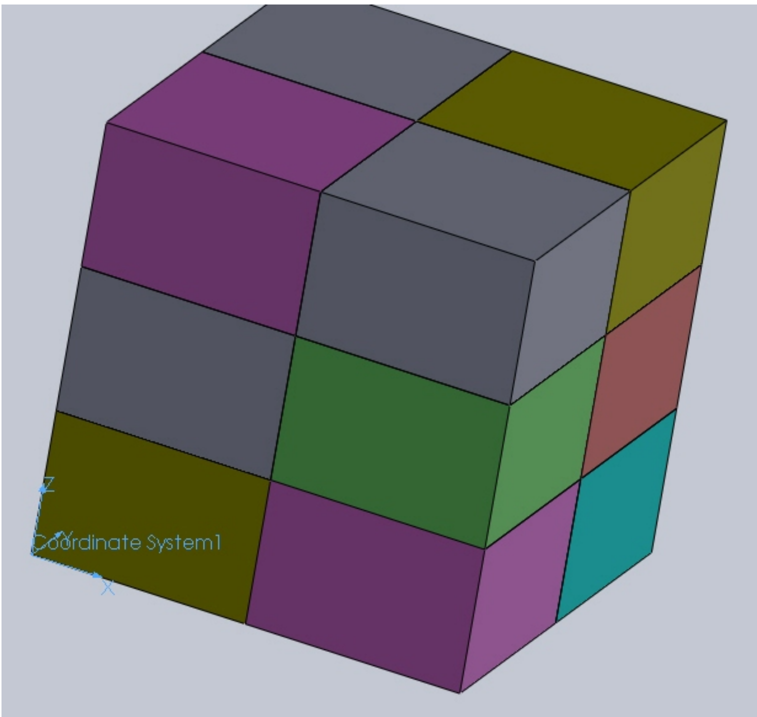

Figure 3.2: example of dividing furnace enclosure into several zones

The alignment of coordinate system can be done by choosing z = 0 plane, y = 0 planeand x = 0 plane. All other furnace walls can be represented by using this coordinate system and applying the length, width and height dimensions to y, x, z planes respectively. Having defined the coordinate system, vector algebra application for Eq. 2.14, 2.15, 2.16 can be continued. Suppose bottom surface is i and left surface is j.

According to eq. 2.10 $\hat{n}_i = \hat{k}$ and $\hat{n}_j = \hat{\imath}$ for z=0 and x=0 planes. With these normal vector values 2.12a and 2.12b can be determined as

$$cos\theta_i = \frac{\hat{n}_i . s_{ij}}{S} = \frac{1}{S}\lfloor(z_j - z_i)\rfloor = \frac{1}{S}\lfloor(z_j - 0)\rfloor$$ Eq. 3.1a

$$cos\theta_j = \frac{\hat{n}_j . s_{ji}}{S} = \frac{1}{S}\lfloor(x_i - x_j)\rfloor = \frac{1}{S}\lfloor(x_i - 0)\rfloor$$ Eq. 3.1b

Furthermore, small area element of z = 0 plane can be given by dAi = dxi.dyi and small area element of x = 0 plane can be given by dAj = dyjdzj. Let the length is to be divided into n1 segments, width is to be divided into n2 segments and height is to be divided into n3 segments. This will define the number of surface zones and number of volume zones for zonal method application. Once n1, n2 and n3 values are defined total number of surface zones in rectangular surface will be (n1 x n2 + n2 x n3 + n3 x n1) x 2. Total number of volume zones will be n1 x n2 x n3

With the aid of all above information, the equation for DEA between one surface zone of the bottom surface into another surface zone of left surface can be transformed into following format for bottom to left surface.

$$\overline{s_i s_j} = \int_{xi1}^{xi2}\int_{yi1}^{yi2}\int_{yj1}^{yj2}\int_{zj1}^{zj2} e^{-ks}\frac{z_j x_i}{\pi S^4} dz_j\, dy_j\, dy_i\, dx_i$$ Eq. 3.2a

Where $S = \left(x_i^2 + \left(y_j - y_i\right)^2 + z_j^2\right)^{1/2}$ and k value should be provided as an input. Furthermore, the difference between the upper limit and the lower limit of the integral represents the step size or zone size along that particular axis.

Similar fashion can be followed to obtain the equation for the DEA between top surface to left surface. Due to the similarity, the equation will be look like same. However, the boundary condition of the furnace wall will be represented in the equation with respect to the coordinate system.

$$\overline{s_i s_j} = \int_{xi1}^{xi2} \int_{yi1}^{yi2} \int_{yj1}^{yj2} \int_{zj1}^{zj2} e^{-ks} \frac{(height - z_j)x_i}{\pi S^4} dz_j \, dy_j \, dy_i \, dx_i \quad \text{Eq. 3.2b}$$

Since, the rectangular furnaces are axis symmetry, obtaining the DEAs between bottom surface to left surface will sufficient to obtain the respect values for top to left surface with using the concept of mirror image. However, the coordinate values will be changed.

Reciprocity relation of DEAs will reduce the amount of effort which has to be done on repeating the calculations. Having considered these factors, it can be identified that there will be 12 such relations in the format of equation 3.2a and 3.2b to represent all perpendicular surfaces in a rectangular furnace. There will be 3 other relations to represent parallel surface to surface exchange areas. By applying the same concept which is shown in Eq. 3.2a integral form can be developed for top to bottom surfaces.

$$\overline{s_i s_j} = \int_{xi1}^{xi2} \int_{yi1}^{yi2} \int_{yj1}^{yj2} \int_{xj1}^{xj2} e^{-ks} \frac{height^2}{\pi S^4} dx_j \, dy_j \, dy_i \, dx_i \quad \text{Eq. 3.3}$$

Similarly, other surface to surface parallel exchange area relations can be obtained. With the set of equations in the format of 3.2 and 3.3 all fundamental integral form to determine surface to surface DEAs can be identified.

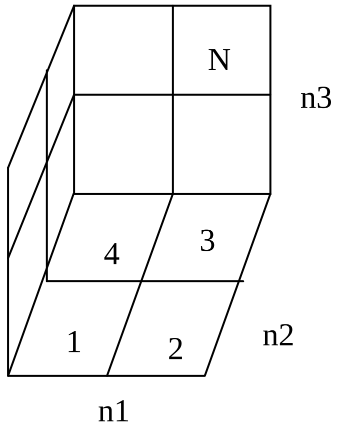

Figure 3.3: surface zone labeling in a furnace enclosure

Observing figure 3.2, it can be seen that N x N number of DEAs should be found theoretically to represent all surface to surface DEA factors. Due to area factors in between zones in same plane zero, the number of determinations will be reduced.

By applying same kind of mathematical treatment on Eq. 2.15 which can be converted into following integral form.
For bottom surface zone into volume zone

$$\overline{g_i s_j} = \int_{zi1}^{zi2} \int_{yi1}^{yi2} \int_{xi1}^{xi2} \int_{yj1}^{yj2} \int_{xj1}^{xj2} e^{-ks} \frac{z_i}{\pi S^3} k dx_j \, dy_j \, dx_i \, dy_i dz_i \qquad \text{Eq. 3.4}$$

If there are N number of surface zones and k number of volume zones in the enclosure, then there will be a N x K matrix which has to be determined.
For volume to volume zones

$$\overline{g_i g_j} = \int_{zi1}^{zi2} \int_{yi1}^{yi2} \int_{xi1}^{xi2} \int_{zj1}^{zj2} \int_{yj1}^{yj2} \int_{xj1}^{xj2} e^{-ks} \frac{k^2}{\pi S^2} dx_j \, dy_j \, dz_j \, dx_i \, dy_i dz_i \qquad \text{Eq. 3.5}$$

If there are K number of volume zones available in the enclosure. Then there will be a K x K matrix which has to be determined.

Although, the integral form in equation 2.14, 2.15 and 2.16 is simplified for a furnace (Eq. 3.2, 3.3 and 3.4), still analytical evaluation is a tedious task since evaluation includes above four integrals. Even software focused on mathematical calculations find it difficult to estimate at least up to second exponential integral.

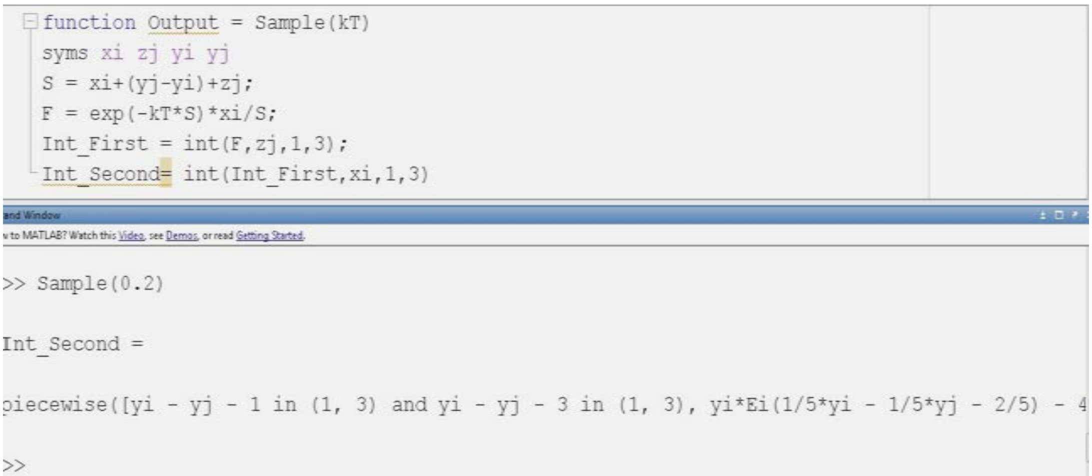

Figure 3.4: estimation of a sample of equation 3.2 usingMatlab

Figure 3.3 shows how much it is difficult for evenMatlab software to estimatesimpler version of required formula.

A method for reducing double integrals in to two single integrals has been performed [11] . According to that

$$I = \int_0^a \int_0^b f(x-y)dxdy = \int_0^a zf(z-a)dydz + a\int_a^b f(z-a)dz + \int_b^{a+b}(a+b-z)f(z-a)dz \qquad \text{Eq. 3.6}$$

By repeating Eq. 2.9

$$s_{ij} = (x_j - x_i)\hat{\imath} + (y_j - y_i)\hat{\jmath} + (z_j - z_i)\hat{k}$$

A function based on s_{ij} can be converted in to Eq. 3.6 format. In a furnace when two perpendicular surfaces are considered, always one axis will be common to both surfaces. That part can be reduced with Eq. 3.6 scheme and reduced in to triple integral rather than quadruple integral.

By repeating Eq. 3.2a

$$\overline{s_i s_j} = \int_{xi\,1}^{xi\,2}\int_{yi\,1}^{yi\,2}\int_{yj\,1}^{yj\,2}\int_{zj\,1}^{zj\,2} e^{-ks}\,\frac{z_j x_i}{\pi S^4}\,dz_j\,dy_j\,dy_i\,dx_i$$

can be converted to

$$\overline{s_i s_j} = \int_{xi\,1}^{xi\,1+\Delta xi}\int_{yi\,1}^{yi\,1+\Delta yi}\int_{yj\,1}^{yj\,1+\Delta yj}\int_{zj\,1}^{zj\,1+\Delta zj} e^{-ks}\,\frac{z_j x_i}{\pi S^4}\,dz_j\,dy_j\,dy_i\,dx_i \qquad \text{Eq. 3.7}$$

Where Δ values represent the step sizes of the zone along each axis. By variable transformation this can be converted to

$$\overline{s_i s_j} = \int_{0}^{\Delta xi}\int_{0}^{\Delta yi}\int_{0}^{\Delta yj}\int_{0}^{\Delta zj} e^{-ks}\,\frac{z_2 x_1}{\pi S^4}\,dz_2\,dy_2\,dy_1\,dx_1 \qquad \text{Eq. 3.8}$$

Where

$$x_1 = x_i - x_{i1}$$

$$y_1 = y_i - y_{i1}$$

$$y_2 = y_j - y_{j1}$$

$$z_2 = z_j - z_{j1}$$

$$S = \left((x_{i1} + x_1)^2 + \left(y_{j1} - y_{i1} + y_2 - y_1\right)^2 + \left(z_{j1} + z_2\right)^2\right)^{1/2} [12]$$

This can be converted by using Eq 3.6

$$\overline{s_i s_j} = \int_{0}^{\Delta zj}\int_{0}^{\Delta xi}\left[\int_{-\Delta yi}^{0}(\Delta y_i + y)\,e^{-ks}\,\frac{z_2 x_1}{\pi S^4}\,\mathrm{dy} + \int_{0}^{\Delta yj}(\Delta y_j - y)e^{-ks}\,\frac{z_2 x_1}{\pi S^4}\,\mathrm{dy}\right] dx_1\,dz_2$$

Eq. 3.9

Where y = $y_2 - y_1$

By repeating Eq. 3.3 for parallel planes

$$\overline{s_i s_j} = \int_{xi1}^{xi2}\int_{yi1}^{yi2}\int_{yj1}^{yj2}\int_{xj1}^{xj2} e^{-ks}\,\frac{height^2}{\pi S^4}\,dx_j\,dy_j\,dy_i\,dx_i$$

And

$$S = \left((x_2 - \mathrm{x}_1 + x_{j1} - x_{\mathrm{i}1})^2 + (y_{j1} - y_{i1} + y_2 - y_1)^2 + (\mathrm{height})^2\right)^{1/2}$$

In this case it can be seen that two common axis exists for the planes. Therefore quadruple integral can be reduced in to double integral. By using similar kind of transformations used in Eq. 3.7, 3.8 and 3.9, the resulting equation would be

$$\overline{s_i s_j} = \int_{-\Delta\mathrm{xi}}^{0}\int_{-\Delta\mathrm{yi}}^{0}(\Delta\mathrm{yi} + \mathrm{y})(\Delta\mathrm{xi} + \mathrm{x})\, e^{-ks}\,\frac{height^2}{\pi S^4}\,\mathrm{dxdy} + \int_{0}^{\Delta\mathrm{xj}}\int_{-\Delta\mathrm{yi}}^{0}(\Delta\mathrm{yi} + \mathrm{y})(\Delta\mathrm{xi} - \mathrm{x})\, e^{-ks}\,\frac{height^2}{\pi S^4}\,\mathrm{dxdy} + \int_{-\Delta\mathrm{xi}}^{0}\int_{0}^{\Delta\mathrm{yj}}(\Delta\mathrm{yi} - \mathrm{y})(\Delta\mathrm{xi} + \mathrm{x})\, e^{-ks}\,\frac{height^2}{\pi S^4}\,\mathrm{dxdy} + \int_{0}^{\Delta\mathrm{xj}}\int_{0}^{\Delta\mathrm{yj}}(\Delta\mathrm{yi} - \mathrm{y})(\Delta\mathrm{xi} - \mathrm{x})\, e^{-ks}\,\frac{height^2}{\pi S^4}\,\mathrm{dxdy}$$

Eq. 3.10

By repeating Eq. 3.4 for volume to surface zone

$$\overline{g_i s_j} = \int_{zi1}^{zi2}\int_{yi1}^{yi2}\int_{xi1}^{xi2}\int_{yj1}^{yj2}\int_{xj1}^{xj2} e^{-ks}\,\frac{z_i}{\pi S^3}\,k\,dx_j\,dy_j\,dx_i\,dy_i\,dz_i$$

And

$$S = \left((x_2 - \mathrm{x}_1 + x_{j1} - x_{\mathrm{i}1})^2 + (y_{j1} - y_{i1} + y_2 - y_1)^2 + (\mathrm{z}_1 + \mathrm{z}_{\mathrm{i}1})^2\right)^{1/2}$$

In this case, similar to Eq. 3.10 two axis will be parallel. However, one additional axis will remain. Therefore, similar pattern can be expected with additional integral.

$$\overline{g_i s_j} = \int_0^{\Delta zi}\Big[\int_{-\Delta xi}^{0}\int_{-\Delta yi}^{0}(\Delta yi + y)(\Delta xi + x)\, e^{-ks}\,\frac{z_i}{\pi S^3}\,\text{kdxdy} + \int_0^{\Delta xj}\int_{-\Delta yi}^{0}(\Delta yi + y)(\Delta xi - x)\, e^{-ks}\,\frac{z_i}{\pi S^3}\text{kdxdy} + \int_{-\Delta xi}^{0}\int_0^{\Delta yj}(\Delta yi - y)(\Delta xi + x)\, e^{-ks}\,\frac{z_i}{\pi S^3}\,\text{kdxdy} + \int_0^{\Delta xj}\int_0^{\Delta yj}(\Delta yi - y)\, x)\, e^{-ks}\,\frac{z_i}{\pi S^3}\,\text{kdxdy}\Big]\,\text{dz1}$$

Eq. 3.11

By repeating Eq. 3.5

$$\overline{g_i g_j} = \int_{zi1}^{zi2}\int_{yi1}^{yi2}\int_{xi1}^{xi2}\int_{zj1}^{zj2}\int_{yj1}^{yj2}\int_{xj1}^{xj2} e^{-ks}\,\frac{k^2}{\pi S^2}\,dx_j\,dy_j\,dz_j\,dx_i\,dy_i\,dz_i$$

Following the same style as above

$$\begin{aligned}\overline{g_i g_j} = {} & \int_{-\Delta xi}^{0}\int_{-\Delta yi}^{0}\int_{-\Delta zi}^{0}(\Delta xi + x)(\Delta yi + y)\,(\Delta zi + z)\, e^{-ks}\,\frac{k^2}{\pi S^2}\,\text{dzdydz} + \\ & \int_0^{\Delta xj}\int_{-\Delta yi}^{0}\int_{-\Delta zi}^{0}(\Delta xi - x)(\Delta yi + y)\,(\Delta zi + z)\, e^{-ks}\,\frac{k^2}{\pi S^2}\text{dzdydx} + \\ & \int_{-\Delta xi}^{0}\int_0^{\Delta yj}\int_{-\Delta zi}^{0}(\Delta xi + x)(\Delta yi - y)\,(\Delta zi + z)\, e^{-ks}\,\frac{k^2}{\pi S^2}\,\text{dzdydx} + \\ & \int_0^{\Delta xj}\int_0^{\Delta yj}\int_{-\Delta zi}^{0}(\Delta xi - x)\,(\Delta yi - y)(\Delta zi + z)\, e^{-ks}\,\frac{k^2}{\pi S^2}\,\text{dzdydx} + \\ & \int_{-\Delta xi}^{0}\int_{-\Delta yi}^{0}\int_0^{\Delta zj}(\Delta xi + x)(\Delta yi + y)\,(\Delta zj - z)\, e^{-ks}\,\frac{k^2}{\pi S^2}\,\text{dzdydz} + \\ & \int_0^{\Delta xj}\int_{-\Delta yi}^{0}\int_{-\Delta zi}^{0}(\Delta xi - x)(\Delta yi + y)\,(\Delta zj - z)\, e^{-ks}\,\frac{k^2}{\pi S^2}\text{dzdydx} + \\ & \int_{-\Delta xi}^{0}\int_0^{\Delta yj}\int_{-\Delta zi}^{0}(\Delta xi + x)(\Delta yi - y)\,(\Delta zj - z)\, e^{-ks}\,\frac{k^2}{\pi S^2}\,\text{dzdydx} + \\ & \int_0^{\Delta xj}\int_0^{\Delta yj}\int_{-\Delta zi}^{0}(\Delta xi - x)\,(\Delta yi - y)(\Delta zj - z)\, e^{-ks}\,\frac{k^2}{\pi S^2}\,\text{dzdydx}\end{aligned}$$

Eq. 3.12

Although amount of integral has been reduced, still the direct evaluation of Eq. 3.9, 3.10, 3.11, 3.12 equations are not possible

For this, Simpson's rule has been applied on the 3.9 to 3.12 equations.
The equation for Simpson rule is as follows

$$\int_a^b f(x)dx \approx \frac{\Delta x}{3}(y_0 + 4y_1 + 2y_2 + \cdots + 4y_{n-1} + y_n)[13] \qquad \text{Eq. 3.13}$$

Although the Simpson's rule has been derived for single variable function, still it is valid for multi variables as long as they behave independently. The accuracy of the result depends on the size of Δx. For each and every variable, Simpson's rule should be applied separately.

After employing numerical techniques, the results obtained should be compared with globally available results to validate its accurateness. Also, the error percentage should be estimated to identify the variation of errors during calculations. However, most of the literatures do not provide values for DEAs. It is well understood the reason that DEA is depend on the area selected by the user. Furthermore, many literatures avoid the participating medium due to its complexity and the absorption and emission is varied with the temperature and pressure. Reasonable amount of accurate checking could be done by neglecting transmission factor and converting area factor in to view factor. Moreover, Eq. 2.20 and 2.19 can be used for cross check for accurateness.

4. RESULTS

4.1 Structure for the program to determine DEAs

In this studyMatlab[14]has been used due to its extensive support on mathematical techniques. The algorithm structure for evaluating result is given below.

- Identify the two surfaces whose area factors need to be evaluated and determine the basic equation by using equation 3.2 obtaining criteria.

- Obtaining the data from the user in to the interface. Furnace dimensions (length, width, height), number of divisions expected (n1, n_2, n_3 later to be align with any of x, y, z axis) and absorption coefficient variable.

- Defining the coordinate system and align with three edges. Align with bottom left corner would be easy to avoid the issues which might come when the coordinates become negative.

- Dividing furnace dimension by number of divisions to obtain number of zones expected for the model. This can be considered as the step size of the model.

- Determine the coordinates of the corners of the surface zones (four corners). All the coordinate points of surface zone edges can be obtained through an iteration procedure using the step sizes along each axis.

- During each and every iteration, there should be another set of iterations to obtain the coordinate points of the other surface. (each and every surface zone is interacting with each and every other surface zone of the wall)

- Once a set of coordinates of two particular surface zones are found during the iteration from both surfaces, Simpson's rule can be applied for modified equation 3.9, with the limits as the coordinates of the surface zone, to evaluate the direct area exchange.

As mentioned in above, for each and every set of coordinate, there should be separate application of Simpson's ruleduring the evaluation.

This should be continued until the complete wall is covered by surface zones.This procedure is valid for both perpendicular and parallel surfaces.There is no significant difference between estimating surface to surface DEAs and other two situations (surface to gas and gas to gas) except the change of number of integrals. Therefore, the number of coordinates and number of iterations will be increased.

Appendix 2 will be illustrating such a program to obtain DEA between bottom surface and left surface. However, the author has used "triplequad" command in matlab, which is used to perform triple integration using numerical techniques for simplicity. Performing the evaluation using Simpson's rule has been provided in appendix 3 as an comment for readers who wish to perform this evaluation using other software. The basic structure of the program has been modified such a way that estimated coordinate points of surfaces and the estimated DEA value to be stored in an excel file for the data to be used.

4.2 Validation

Following table illustrates such example of bottom to left zones. The inputs are length 2m, width 2m, height 2m, number of zones required in each axis is 2.The reference values for the comparison, were obtained from web page, http://www.thermalradiation.net/calc/sectionc/C-15.html [15].

Table 4.1: view factor between bottom to left surfaces of a cubical furnace of 2 m

X0	X1	Y0	Y1	EP0	EP1	NE0	NE1	V.F from other ways	V.F from software calculation	Error %
0	1	0	1	0	1	0	1	0.2000	0.2000	0.00
0	1	0	1	1	2	0	1	0.0406	0.0406	0.00
0	1	0	1	0	1	1	2	0.0328	0.0328	0.00
0	1	0	1	1	2	1	2	0.0189	0.0189	0.00
1	2	0	1	0	1	0	1	0.0328	0.0328	0.00
1	2	0	1	1	2	0	1	0.0189	0.0189	0.00
1	2	0	1	0	1	1	2	0.0329	0.0329	0.00
1	2	0	1	1	2	1	2	0.0230	0.0230	0.00
0	1	1	2	0	1	0	1	0.0406	0.0406	0.00
0	1	1	2	1	2	0	1	0.2000	0.2000	0.00
0	1	1	2	0	1	1	2	0.0189	0.0189	0.00
0	1	1	2	1	2	1	2	0.0328	0.0328	0.00
1	2	1	2	0	1	0	1	0.0189	0.0189	0.00
1	2	1	2	1	2	0	1	0.0328	0.0328	0.00
1	2	1	2	0	1	1	2	0.0230	0.0230	0.00
1	2	1	2	1	2	1	2	0.0329	0.0329	0.00

With the comparison it can be observed that, this procedure is having maximum accuracy up to 0.0001 for all cases.For the comparison, it has been taken up to fourth decimal place. If there is a necessity to go deeply, result data type should be changed.

Same calculation was performed for parallel surfaces and the result is attached in appendix 3. It can be observed that the error percentage of the calculated result is zero for every case with respect to considered reference.

Considering transmission factor as 0.5, furnace dimension 2m, 2m, 2m and number of segments along each axis is 2, the resulting data will be stored in 15 spread sheets. Such example of obtainingDEA values bottom left surface zone to other zones given under Table 4.2

Table 4.2: DEA values for bottom, left, front corner surface all 24 surfaces zones

Surface	X0	X1	Y0	Y1	Z0	Z1	DEA
Left	0	0	0	1	0	1	0
Left	0	0	1	2	0	1	0
Left	0	0	0	1	1	2	0
Left	0	0	1	2	1	2	0
Right	2	2	0	1	0	1	0.0243
Right	2	2	1	2	0	1	0.0157
Right	2	2	0	1	1	2	0.0157
Right	2	2	1	2	1	2	0.0105
Bottom	0	1	0	1	0	0	0.1539
Bottom	1	2	0	1	0	0	0.0151

Table 4.2 (Contd.)

Surface	X0	X1	Y0	Y1	Z0	Z1	DEA
Bottom	0	1	1	2	0	0	0.0252
Bottom	1	2	1	2	0	0	0.0078
Front	0	1	0	0	0	1	0.1539
Front	1	2	0	0	0	1	0.0151
Front	0	1	0	0	1	2	0.0252
Front	1	2	0	0	1	2	0.0078
Back	0	1	2	2	0	1	0.0151
Back	1	2	2	2	0	1	0.0117
Back	0	1	2	2	1	2	0.0078
Back	1	2	2	2	1	2	0.0074
Top	0	1	0	1	0	0	0.0151
Top	1	2	0	1	0	0	0.0117
Top	0	1	1	2	0	0	0.0078
Top	1	2	1	2	0	0	0.0074
Total							0.5542

DEA values for front, left, and bottom volume zone to surface zones with transmission factor equals to 0.5 have been given in Table 4.3. Coordinate of the volume zone would be [0 1 0 1 0 1] for all x, y, z axes respectively.

Table 4.3: DEA values for bottom, left, front corner volume all 24 surfaces zones

Surface	X0	X1	Y0	Y1	Z0	Z1	DEA
Left	0	0	0	1	0	1	0.2653
Left	0	0	1	2	0	1	0.0437
Left	0	0	0	1	1	2	0.0436

Table 4.3 (Contd.)

Surface	X0	X1	Y0	Y1	Z0	Z1	DEA
Left	0	0	1	2	1	2	0.0141
Right	1	2	0	1	0	1	0.0301
Right	1	2	1	2	0	1	0.0168
Right	1	2	0	1	1	2	0.0168
Right	1	2	1	2	1	2	0.0103
Bottom	0	1	0	1	0	0	0.2349
Bottom	1	2	0	1	0	0	0.0434
Bottom	0	1	1	2	0	0	0.0434
Bottom	1	2	1	2	0	0	0.0141
Front	0	1	0	0	0	1	0.2349
Front	1	2	0	0	0	1	0.0434
Front	0	1	0	0	1	2	0.0434
Front	1	2	0	0	1	2	0.0141
Back	0	1	2	2	0	1	0.0301
Back	1	2	2	2	0	1	0.0168
Back	0	1	2	2	1	2	0.0168
Back	1	2	2	2	1	2	0.0103
Top	0	1	0	1	0	0	0.0301
Top	1	2	0	1	0	0	0.0168
Top	0	1	1	2	0	0	0.0168
Top	1	2	1	2	0	0	0.0103
Total							1.3254

DEA values for front, left, and bottom volume zone to other volume zones with transmission factor 0.5 have been given in Table 4.4.

Table 4.2: DEA values for bottom, left, front corner volume all 8 volume zones

Volume zone number	DEA
1	0.3706
2	0.0641
3	0.0641
4	0.0249
5	0.0640
6	0.0243
7	0.0243
8	0.0131
Total	0.6494

According to Eq. 2.20

$$4kV_i = \sum_{m=1}^{k} \overline{g_i g_m} + \sum_{j=1}^{N} \overline{s_j g_i}$$

Left side vale is equal to 2 (4 x 0.5 x 1). Meanwhile, right hand side sums up to 1.9746. Error percentage of the total calculation is equal to 1.27% ((2-1.9746)/2 x 100%)

Although the validation has been performed for k = 0.5, it is necessary to find out whether the result holds its accuracy up to a reasonable level.

Table 4.5: Error percentages for different k values

K value	Calculated value	Theoretical value	Error %
0.1	0.3967	0.4	0.8250
0.2	0.7921	0.8	0.9875
0.3	1.1870	1.2	1.0833
0.4	1.5808	1.6	1.200
0.5	1.9746	2.0	1.2700
0.6	2.3663	2.4	1.4042
0.7	2.7585	2.8	1.4821
0.8	3.1492	3.2	1.5875
0.9	3.5387	3.6	1.7028

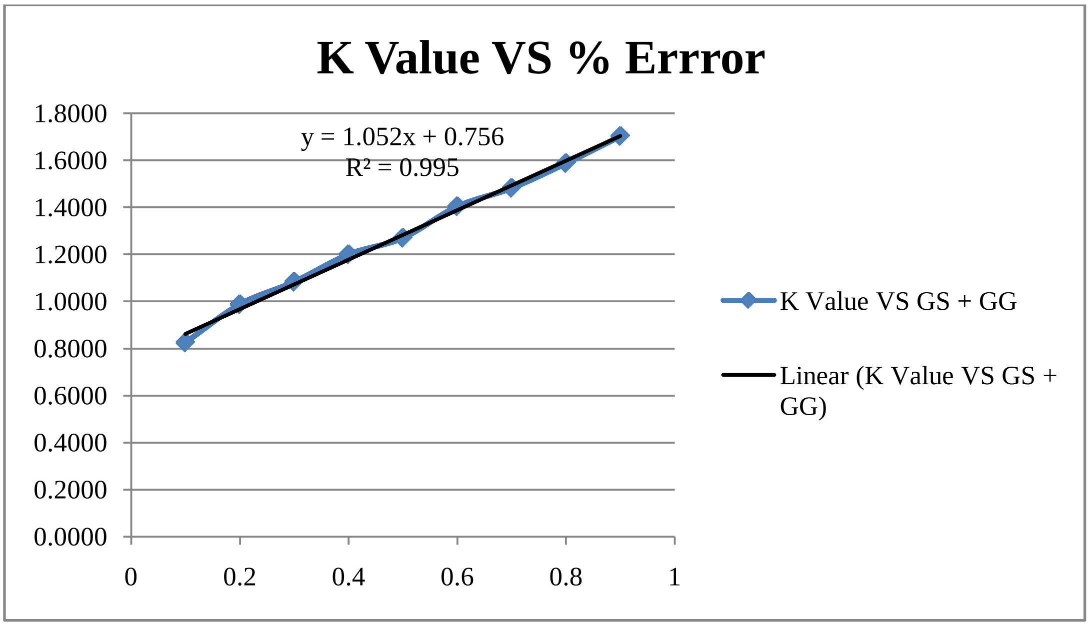

Figure 4.1: K value vs. percentage error

From the plot it can be seen that when absorption coefficient is increased, then the model accuracy will decreased. However, still the result maintains more than 95% accurateness.

Further improvements of the developed program structure were done to hide all the informative programs where the designer may not interested. A software interface shown in figure 4.2, has been developed to serve the above purpose.

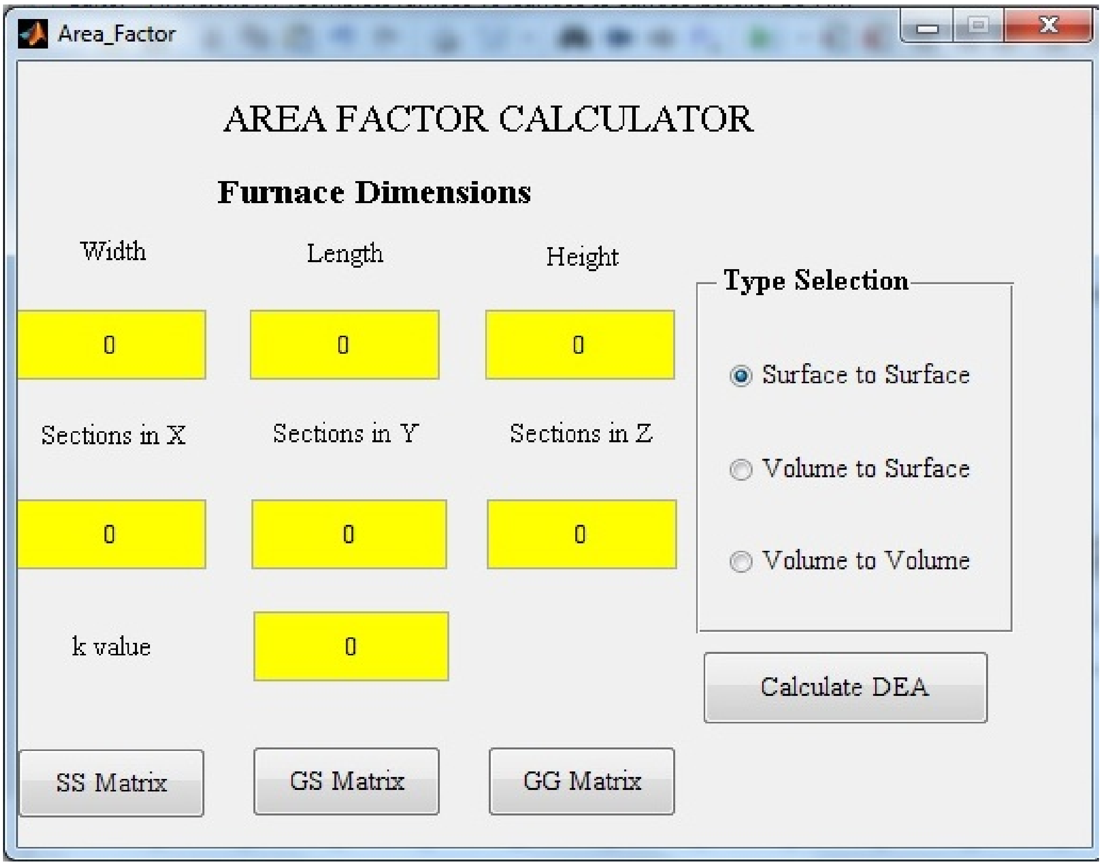

Figure 4.2: Area factor calculator software interface

The user needs to add relevant data into the software, which is based on matlab and press calculate button. Then the software will automatically calculate the DEA values and stored in excel files depend upon selection of the surface to surface or volume to surface or volume to volume selection. Once the data calculation is done and by pressing SS Matrix or GS Matrix of GG Matrix buttons, results for Eq. 2.21 or 2.22 or 2.23 will be generated. This result can be directly used for further calculation of radiation heat transfer.

5. DISCUSSION

Estimation of view factor is a difficult task due to involvement of complicated mathematics.Many literature can be found as graphical representation shown in appendix 5 or online calculators. In the real scenario where the participating medium comes to play, DEA should be estimated.Equations for estimate the DEA values have been introduced[10]. Further extension of these equations to represent rectangular enclosures using zonal method has been introduced under the same work. However, how the evaluation is being carried out when it contains exponential integral has not been developed under that work. The introduced set of equations requires quadruple integral for surface to surface DEA values, quintuple integrals for volume to surface DEA values and sextuple integrals for volume to volume DEA values. This needs an extensive of work plus higher time consumption for evaluation. Reducing of integration when the axis becomes parallel or common between two zones has been introduced[11]. The technique has been further extended to find DEAs for cylindrical enclosure[12]for certain scenarios. The ultimate requirement for the designer is to use results based on his requirements to estimate radiation heat transfer using zonal method and focus on other heat transferring modes rather than spending time and effort on estimating DEA value estimation.The developed software interface focus to aid furnace designers and performance evaluators to make judgments on radiation heat transfer in the system using evaluated DEA values.

The program interface itself is having several features. The user has the freedom to alter the enclosure size according to his requirement. This can be done by varying first three inputs, length, width and height values. Next three inputs (n1, n2, n3) where zone size is determined, will enable the user to define his expected isothermal zones. By giving larger number for n1, n2, n3, higher number of zones can be achieved with higher calculation time. Obtaining higher number of zones will enable the user to introduce various features such as heat generation, heat leakage to the system for each zone while maintaining isothermal zone concept. However, introducing large number of zones into the system might not be effective due to two reasons. Firstly, during the simple example it can be observed that closer segments have higher DEA value than far segments. Therefore, DEA value for far segment might become zero for many cases on the basis of decimal accuracy. Secondly, closer volume segments have self-irradiation, which will leads for singularity generation in DEA estimation. Optimized number of zones needed to be identified by the user considering his requirements.

Once the result is obtained, next step of irradiation estimation can be continued. The added advantage of estimating DEA is to get an overall pictureregarding the heat transfer performance of that enclosure. For an example, if simplest version of DEA, view factor is considered, higher the view factor means higher possibility to transfer heat between both surfaces. The amount will be positive or negative with respect to the temperatures of the surfaces. Let us assume ith gas volume zone is having a heat generation source (eg: a burner). If s_1s_i is higher (may be greater than 1 depending on the area) with respective to other, then it will be an indication there will be efficient heat transfer to s_1 surface and it might have a higher temperature. By observing the DEA values one can predict the suitable location for the work piece, if the furnace is already available. If not, burner or heat input locations can be analyzed to achieve a uniform heat input to the object.

Results for the simplest version of DEA, view factor has been proven with well established data.Moreover, Eq. 2.20 indicates that the total result consists of an error about 1.27%. This error basically comes due to self-irradiation of volume zones. Another possibility is the error incurred in numerical techniques. It is needed to be found that which plays the majority role and apply suitable solution to improve accuracy. Furthermore, it has been shown in figure 4.1 that the accuracy of the obtained results deviate more and more when k value rises. Although the correlation is identified for different k values, it is necessary to find out the ratio of volume to volume zones and volume to surface zone contribution to the generated error.

As defined in literature review, this structure is to cater for rectangular enclosures whose surface materials and participating media are having non scattering, grey and isotropic features. Further investigating should be done to include above features and included in to the software interface to provide much flexibility for the consumer. Performing DEA evaluation in the presence of obstruct can be suggested for a future work. However, according to table 2.1 and figure 2.2 it can be seen that when all these issues arises the problem becomes much complicated and better to move for Monte Carlo technique.

6. CONCLUSION

For a furnace whose operating temperature is above 1000^0C, it is essential to develop radiation heat transfer model to understand the performance of the furnace. Obtaining radiation heat transfer model using zonal method requires DEA values for each and every zone. Such difficult task for rectilinear box shape furnace could be achieved by applying vector algebra and reduction integration scheme to general equations for the complete enclosure and simplify the resulting equations by using Simpson’s rule. Resulting computer program can handle rectilinear box shapes with any dimension with any transmission factor. Depend upon the accuracy requirement and the time constraint, the user is provided with the privilege to alter the zone size.

The validation is done by using basic form of DEA (Viewfactor) and comparing the result with mathematically proven equation. Ways of reducing the error percentage is discussed in order to achieve better results is suggested for future work.

Extension of this program to fine tune for garbage inputs from users, optimum amount of zones for reasonable accuracy, reducing time consumption has been suggested for further improvements.

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8. APPENDICES

Appendix 1: Emissivity values of different gases at different temperature and pressures

[2]

Table 1. Summary of database for the emissivity of pure gases.

	Number of Data Points	Emissivity (ε)		Temperature *(T)*, K		*pL*, kPa•m	
		Maximum	Minimum	Maximum	Minimum	Maximum	Minimum
CO_2	939	2,788	274	154	0.031	0.23	3.0×10^{-3}
H_2O	1,034	2,776	278	616	0.15	0.66	7.0×10^{-3}
CO	425	1,333	334	62	0.12	0.12	5.0×10^{-3}
CH_4	447	2,088	287	62	0.31	0.24	1.0×10^{-2}
SO_2	394	1,948	396	62	0.062	0.46	3.0×10^{-3}
NH_3	300	1,446	274	62	0.022	0.72	2.0×10^{-3}

Table 2. Summary of database for the emissivity correction factors C_{H_2O}, C_{CO_2}, and C_{SO}.

	Number of Data Points	Pressure *(P)*, kPa		*pL*, kPa•m		Correction Factor	
		Maximum	Minimum	Maximum	Minimum	Maximum	Minimum
C_{H_2O}	151	122	0	309	0	1.7	0.1
C_{CO_2}	190	507	5	77	0	1.7	0.3
C_{SO}	288	N.A.	N.A.	62	3	0.08	0.0

Appendix 2: Sample DEA calculation for bottom to left surface

```
function Output = perpendicular_Bo_L(length,width,height,n1,n2,n3,kT)

% Obtaining the user inputs

count = 0;

stepX = width/n1;
stepY = length/n2;
stepZ = height/n3;

% Defining the step size

start_x = 0;
start_y = 0;
start_z = 0;




for j = 1:n2
for i = 1:n1
pt_y(j).pt_x(i).z_0 = [ start_x, start_y, start_z;start_x, start_y+stepY, start_z;start_x+stepX, start_y+stepY, start_z;start_x+stepX,
start_y, start_z ];


    A = pt_y(j).pt_x(i).z_0; % calculating pointts of surface zones for bottom surface Z = 0.
start_x = start_x+stepX;
```

```
end
start_x = 0;
start_y = start_y+stepY;
end


start_x = 0;
start_y = 0;
start_z = 0;


%
for k = 1:n3
for j = 1:n2
pt_z(k).pt_y(j).x_0 = [ start_x, start_y, start_z;start_x, start_y+stepY, start_z;start_x, start_y+stepY, start_z+stepZ;start_x, start_y,
start_z+stepZ ];



    B = pt_z(k).pt_y(j).x_0; % calculating pointts of surface zones for left surface
start_y = start_y+stepY;

end
start_y = 0;
start_z = start_z+stepZ;
end
```

```
% Following procedure uptosheetnum 1 was used to open an excel file to
% record the values


currentfile = mfilename('fullpath');
 [pathstr,name,ext] = fileparts(currentfile);
 S = fullfile(pathstr,'surface2surface.xls')
addpath(pathstr)
file = S;
 Excel = actxserver('Excel.Application');
 Workbooks = Excel.Workbooks;
Excel.Visible=1;
 Workbook=Workbooks.Open(file)
sheetnum=8;


%
for j = 1:n2

for i = 1:n1
x0 = pt_y(j).pt_x(i).z_0(1,1);
x1 = pt_y(j).pt_x(i).z_0(3,1);
       y0 = pt_y(j).pt_x(i).z_0(1,2);
       y1 = pt_y(j).pt_x(i).z_0(2,2);
       z0 = pt_y(j).pt_x(i).z_0(1,3);
       z1 = pt_y(j).pt_x(i).z_0(3,3);

      A=  [x0 x1, y0 y1];
```

```
for k = 1:n3
for l = 1:n2

            ep0= pt_z(k).pt_y(l).x_0(1,2);
            ep1= pt_z(k).pt_y(l).x_0(2,2);
            ne0= pt_z(k).pt_y(l).x_0(1,3);
            ne1= pt_z(k).pt_y(l).x_0(3,3);
            th0= pt_z(k).pt_y(l).x_0(1,1);
            th1= pt_z(k).pt_y(l).x_0(3,1);

          B=   [ep0 ep1, ne0 ne1];

           [Coordinate, Area_Factor] = f(x0,x1,y0,y1,z0,z1,ep0,ep1,ne0,ne1,th0,th1,stepX,stepY,stepZ,kT)

%  Recording the direct exchange area values that has been obtained


count = count+1;
          rn1   = 12+count;
          rn2   = 12+count;
          range1 = sprintf('B%d:M%d',rn1,rn1);
          range2 = sprintf('O%d:O%d',rn2,rn2);
          Sheets = Excel.ActiveWorkBook.Sheets;
          sheet1 = get(Sheets, 'Item', sheetnum);
invoke(sheet1, 'Activate');
Activesheet = Excel.Activesheet;
ActivesheetRange = get(Activesheet,'Range',range1);
set(ActivesheetRange, 'Value', Coordinate);
          Range = get(Activesheet,'Range',range1);
```

```
out = Range.value;
ActivesheetRange = get(Activesheet,'Range',range2);
set(ActivesheetRange, 'Value',  Area_Factor);
        Range = get(Activesheet,'Range',range2);
out = Range.value;


end

end


end

end

% Saving the opened work book

invoke(Workbook,'Save')
invoke(Excel,'Quit');
delete(Excel);
clearExcel;




function [Coordinate, Area_Factor] = f(x0,x1,y0,y1,z0,z1,ep0,ep1,ne0,ne1,th0,th1,stepX,stepY,stepZ,kT)

symsxyz
```

```
% Lower Limits
xi1 = x0; xj1 = th0 ;
yi1 = y0; yj1 = ep0 ;
zi1 = z0; zj1 = ne0 ;
dxi = stepX; dxj = 0 ;
dyi = stepY; dyj = stepY ;
dzi = 0; dzj = stepZ ;

 F = @(x,y,z)(((stepY+y).*exp(sqrt((xi1+x).^2+(yi1-yj1+y).^2+(zj1+z).^2).*(-kT)).*(xi1+x).*(zj1+z))./((((xi1+x).^2+(yi1-
yj1+y).^2+(zj1+z).^2).^2).*pi));

 G = @(x,y,z)(((stepY-y).*exp(sqrt((xi1+x).^2+(yi1-yj1+y).^2+(zj1+z).^2).*(-kT)).*(xi1+x).*(zj1+z))./((((xi1+x).^2+(yi1-
yj1+y).^2+(zj1+z).^2).^2).*pi));

 Q = triplequad(F,0,dxi,-dyi,0,0,dzj);
 R = triplequad(G,0,dxi,0,dyj,0,dzj);
Coordinate = [x0 x1 y0 y1 z0 z1 th0 th1 ep0 ep1 ne0 ne1];
Area_Factor = Q+R;
```

Appendix3: Sample program which uses Simpson's rule to perform numerical integration

```
% function [Coordinate, Area_Factor] = f(x0,x1,y0,y1,z0,z1,ep0,ep1,ne0,ne1,th0,th1,stepX,stepY,stepZ,kT)
%
%  n=8;
%
%
% i=0;
% j=0;
% k=0;
% l=0;
%
%
%
%
% syms x y z
%
% xi1 = x0; xj1 = th0 ;
% yi1 = y0; yj1 = ep0 ;
% zi1 = z0; zj1 = ne0 ;
% dxi = stepX; dxj = 0 ;
% dyi = stepY; dyj = stepY ;
% dzi = 0; dzj = stepZ ;
%
%
% Base_Function_1 = (stepY+y)*exp(sqrt((xi1+x)^2+(yj1-yi1+y)^2+(zj1+z)^2)*(-kT))*(xi1+x)*(zj1+z)/((((xi1+x)^2+(yi1-
yj1+y)^2+(zj1+z)^2)^2)*pi);
```

```
% Base_Function_2 = (stepY-y)*exp(sqrt((xi1+x)^2+(yj1-yi1+y)^2+(zj1+z)^2)*(-kT))*(xi1+x)*(zj1+z)/((((xi1+x)^2+(yi1-yj1+y)^2+(zj1+z)^2)^2)*pi);
%
% for j = 0:(dzj)/n:dzj
%        r = r+1;
%        Z(r) = j;
%        z = Z(r);
%        Base1F1(r) = eval(Base_Function_1);
%        Base1F2(r) = eval(Base_Function_2);
%
%    end
%
%     First_Int_1 = (dzj)*(Base1F1(1)+ 4*Base1F1(2)+ 2*Base1F1(3) + 4*Base1F1(4)+ 2*Base1F1(5)+ 4*Base1F1(6)+ 2*Base1F1(7)+ 4*Base1F1(8)+Base1F1(9))/(3*n);
%     First_Int_2 = (dzj)*(Base1F2(1)+ 4*Base1F2(2)+ 2*Base1F2(3) + 4*Base1F2(4)+ 2*Base1F2(5)+ 4*Base1F2(6)+ 2*Base1F2(7)+ 4*Base1F2(8)+Base1F2(9))/(3*n);
%
%
%     for i = -dyi:(dyi)/n:0
%        s = s+1;
%        Yi(s) = i;
%        y = Yi(s);
%        Base2F1(s) = eval(First_Int_1);
%     end
%
%    Second_Int_1 = (dyi)*(Base2F1(1)+ 4*Base2F1(2) + 2*Base2F1(3) + 4*Base2F1(4)+ 2*Base2F1(5)+ 4*Base2F1(6)+ 2*Base2F1(7)+ 4*Base2F1(8)+Base2F1(9))/(3*n);
%
%
```

```
%      for k = 0:(dyj)/n:dyj
%         q = q+1;
%         Yk(q) = k;
%         y = Yk(q);
%         Base2F2(q) = eval(First_Int_2);
%
%      end
%
%      Second_Int_2 = (dyj)*(Base2F2(1)+ 4*Base2F2(2) + 2*Base2F2(3) + 4*Base2F2(4)+ 2*Base2F2(5)+ 4*Base2F2(6)+
2*Base2F2(7)+ 4*Base2F2(8)+Base2F2(9))/(3*n);
%
%
%
%     for l = 0:(dxi)/n:dxi
%         p = p+1;
%         Xi(p) = l;
%         x = Xi(p);
%         Base3F1(p) = eval(Second_Int_1);
%         Base3F2(p) = eval(Second_Int_2);
%     end
%
%      Third_Int_1 = (dxi)*(Base3F1(1)+ 4*Base3F1(2) + 2*Base3F1(3) + 4*Base3F1(4)+ 2*Base3F1(5)+ 4*Base3F1(6)+
2*Base3F1(7)+ 4*Base3F1(8)+Base3F1(9))/(3*n);
%      Third_Int_2 = (dxi)*(Base3F2(1)+ 4*Base3F2(2) + 2*Base3F2(3) + 4*Base3F2(4)+ 2*Base3F2(5)+ 4*Base3F2(6)+
2*Base3F2(7)+ 4*Base3F2(8)+Base3F2(9))/(3*n);


%       Coordinate = [x0 x1 y0 y1 z0 z1 th0 th1 ep0 ep1 ne0 ne1];
%      Area_Factor = Third_Int_1+Third_Int_2;
```

Appendix 4: Sample DEA calculation for bottom to top surface

X0	X1	Y0	Y1	EP0	EP1	NE0	NE1	V.F From other ways	V.F from software calculation	Error %
0	1	0	1	0	1	0	1	0.0686	0.0686	0.00
0	1	0	1	1	2	0	1	0.0481	0.0481	0.00
0	1	0	1	0	1	1	2	0.0481	0.0481	0.00
0	1	0	1	1	2	1	2	0.0351	0.0351	0.00
1	2	0	1	0	1	0	1	0.0481	0.0481	0.00
1	2	0	1	1	2	0	1	0.0686	0.0686	0.00
1	2	0	1	0	1	1	2	0.0351	0.0351	0.00
1	2	0	1	1	2	1	2	0.0481	0.0481	0.00
0	1	1	2	0	1	0	1	0.0481	0.0481	0.00
0	1	1	2	1	2	0	1	0.0351	0.0351	0.00
0	1	1	2	0	1	1	2	0.0686	0.0686	0.00
0	1	1	2	1	2	1	2	0.0481	0.0481	0.00
1	2	1	2	0	1	0	1	0.0351	0.0351	0.00
1	2	1	2	1	2	0	1	0.0481	0.0481	0.00
1	2	1	2	0	1	1	2	0.0481	0.0481	0.00
1	2	1	2	1	2	1	2	0.0686	0.0686	0.00

Appendix 5: Graphical representation of view factor for simple geometries

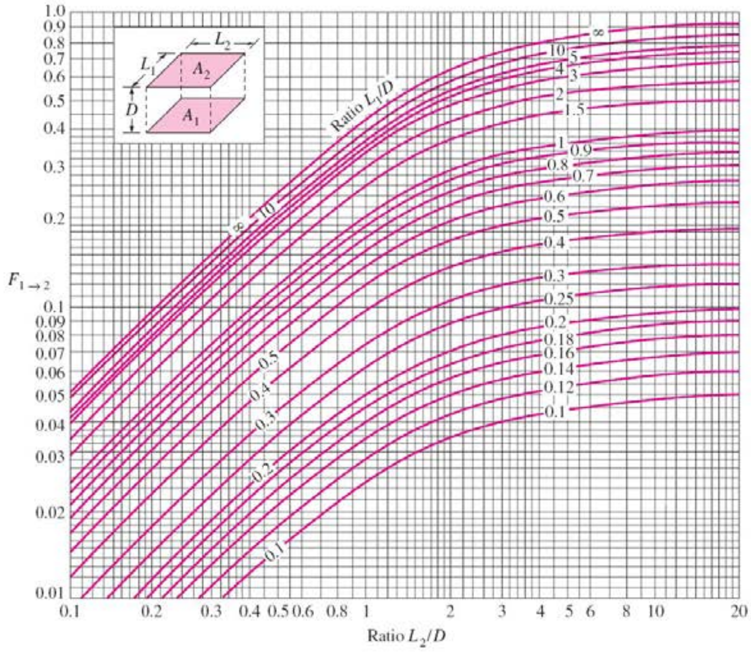

FIGURE 12–5
View factor between two aligned parallel rectangles of equal size.

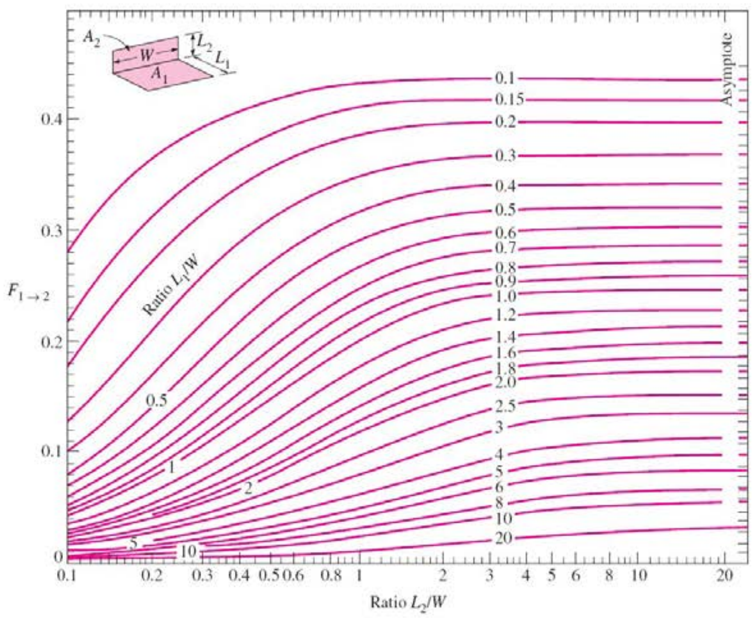

FIGURE 12–6
View factor between two perpendicular rectangles with a common edge.

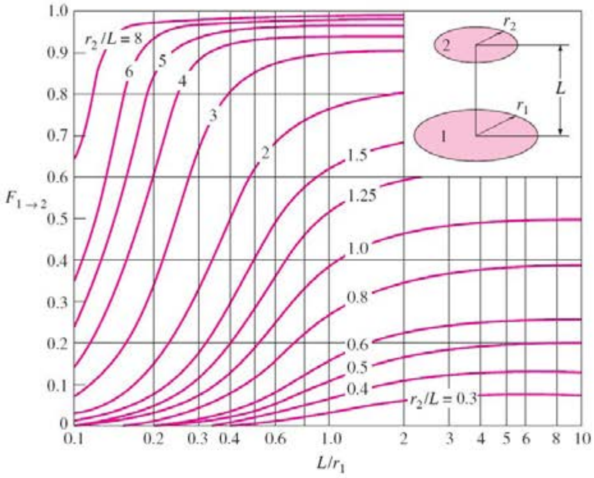

FIGURE 12–7
View factor between two coaxial parallel disks.

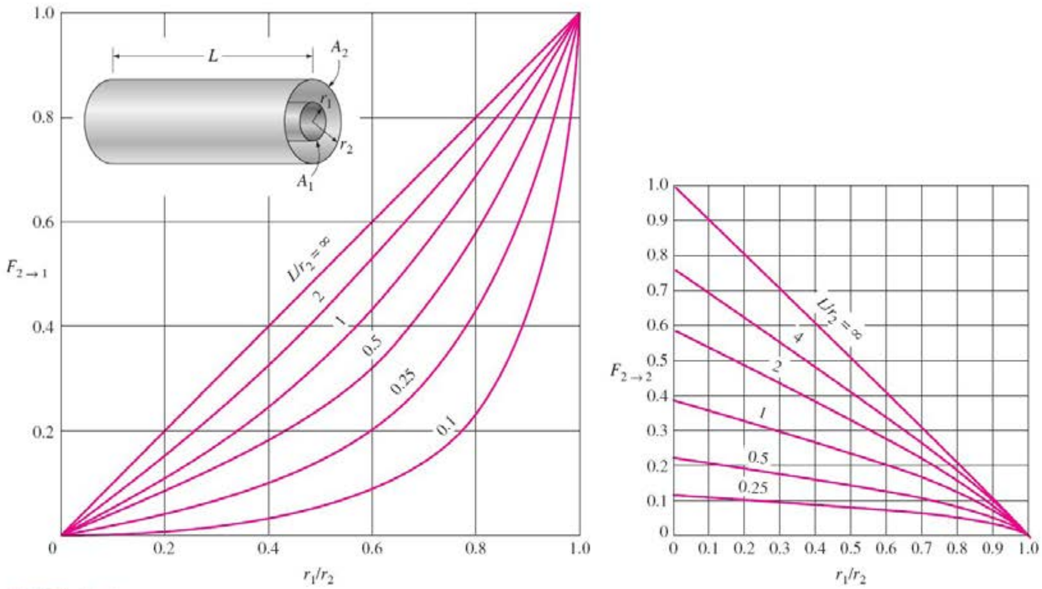

FIGURE 12–8
View factors for two concentric cylinders of finite length: (*a*) outer cylinder to inner cylinder; (*b*) outer cylinder to itself.

Source :[16]