

# Representation of Transformers

## in Switching Surge Studies

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BY

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## ABOUT THE AUTHOR

Born in Colombo, Ceylon, on the 17th October, 1947, the author was educated at St. Joseph's College, Colombo, one of the leading schools in the island.

Because of his extraordinary performance at the Ceylon G.C.E.(Advanced Level) Examination, the author was granted an exemption from following the first year course, and was allowed to complete the four year Electrical Engineering degree in three years.

He obtained First Class Honours in the B.Sc.(Eng), Part I Examination, and completed the degree in August 1969 with Second Class Honours.

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In October 1971, he was awarded a UNESCO Fellowship, and was granted the necessary study leave for post-graduate studies at the University of Manchester Institute of Science and Technology. He obtained the degree of Master of Science in October 1972, and started work on this research project for the degree of Doctor of Philosophy, the findings of which, in the main, are presented in this thesis. His M.Sc. dissertation was on "Cross-bonded Cables and Analysis of Non-linearities".

**To my Parents**



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## Errata

Page iv - Add the following to the list of symbols

- $\bar{i}$  - Winding current (frequency domain)
- $f_n$  - Function chosen for Newton-Raphson algorithm
- $f_n'$  - Jacobian matrix corresponding to chosen function  $f_n$
- $R_v$  - Resistance of non-linear surge divertor at current  $i$
- $S$  - Reluctance values of transformer core and air path
- TF - Time dependant matrices of unit step responses
- U - Unit matrix
- $\phi$  - Flux
- Flux boundaries of non-linear magnetising characteristic
- $v_{gap}$  - Gap voltage in surge divertor
- $t$  - Time increment for numerical integration

Note: Capital letters are used to denote voltages and currents in the frequency domain, while small letters denote them in the time domain, unless otherwise noted.

All variables in chapters 6 & 7 are time dependant variables.

Page 5 - Line 4 : Delete reference '18'

Page 7 - Line 16: Delete 'accurate'

Page 9 - Insert after Line 7

In the non-linear analysis, the remnant magnetism in the transformer core has not been considered and as such the program developed is at present suitable only for the case where the transformer initially has zero remnant magnetism.

Chapter 2 - Mutual coupling  $M_{ij} \neq M_{ji}$  due to the assymetry of core construction. This applies to all mutual coupling terms in the chapter, although otherwise indicated.

Page A-32 - Section A3.2

Line 1 - Replace  $k$  by  $k_{12}$

Line 2 - Replace 'all inductances .....inductance  $L_1$  as' by  
'and if in addition  $k_{21}$  is of the same value, ( $k_{12}=k_{21}=k$ ), then

Insert after Line 5 - This is a reasonable assumption for adjacent windings on same leg.

Insert after Line 11 - However, these relations are no longer accurate when concentric windings or windings on different legs are considered.

Replace page 13. Add pages A-37.2, A37.3 after page A-37.

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ABSTRACT

A Transformer representation suitable for switching surge analysis is derived in terms of 'primitive' inductance and capacitance matrices. Connection matrices are used to account for all types of winding connections. Frequency dependence of elements is considered.

The transmission line is formulated on a frequency basis, and the earth return path is represented by Carson's formula. Series and shunt compensation are also included with the line. The cable is similarly formulated. The problem is solved in the frequency plane using the Fourier Transform, and then transformed back into the time domain. Modal analysis is used for both transmission line and cable two port admittance matrix formulation.

Energisation on both low voltage as well as high voltage sides of the source side transformer are considered for transformer feeders. A few comparisons with field test oscillograms are given to validate the formulation of the problem.

Voltage non-linearities when surge divertors are present are considered using a step response of the linear system and Duhamel's integral, together with the Newton-Raphson iterative technique for both single phase and three operation.

Finally, the Newton-Raphson algorithm is further extended to take into account the magnetic non-linearity of the transformer core. Flux coupling between phases during saturation is calculated based on the instantaneous reluctance values of the individual limbs and the leakage air path.

LIST OF SYMBOLS

The following nomenclature defines the principal symbols generally used. There are instances where the symbols are used locally for different quantities, in which cases they will be defined accordingly.

- A, B - Two port admittance matrix parameters
- C - Capacitance elements
- [C/N] - Matrices to account for types of connection
- E - Voltage source
- $e_i$  - Voltage injected across switch (during switching), phase i
- $f_s$  - Frequency of power system
- G - Conductance of non-linear resistor
- $\underline{I}_i$  - Vector of three phase currents, at position i
- $\underline{V}_i$  - Vector of three phase voltages, at position i  
(double suffix notation is used to denote individual phases)
- $\underline{I}_L$  - Vector of three phase line currents
- J - Notation used for Current source
- k - Coupling factor of flux between primary and secondary
- L - Self inductance of winding
- $l$  - Leakage inductance of winding
- M - Mutual inductance between windings
- n - Secondary/Primary turns ratio
- p - Differential operator, or Transform operator
- R - Notation for resistance
- t - Notation for time
- $v(t), i(t)$  - Voltage and current waveforms in time domain
- $\omega_0$  - Fundamental angular frequency in Fourier analysis
- $\omega$  - Complex angular frequency,

- $[Y]$  - General notation for admittance matrices and submatrices  
(Elements are denoted by double suffix notation)
- $[Y_P]$  - Terminal admittance matrix
- $[Y_S]$  - Source admittance matrix
- $[Y_{tf}]$  - Admittance matrix of transformer
- $[Y_{tL}]$  - Admittance matrix of transmission line
- $[YI]$  - General notation for inverse admittance matrix
- $[Z]$  - Notation for impedance.

